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A Hybrid Control System for an Unstable Non-Stationary Plant with a Predictive Model

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Abstract—A hybrid control system with a discrete adaptive predictive model for a non-stationary unstable third-order dynamic plant in continuous time is synthesized and modeled. An adaptive state observer, estimating a variable parameter of the plant model with respect to the a quadratic quality criterion, was synthesized. Continuous estimation of the plant parameter for a discrete sample is used in a discrete adaptive control algorithm with a predictive model. A linear model of the control plant mimics the unstable vertical motion of plasma in a tokamak with a vertical cross-section elongated along the vertical axis compared to a given equilibrium position.

Keywords: adaptive predictive model, unstable non-stationary plant, adaptive state observer, hybrid system, plasma, tokamak

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1. INTRODUCTION

The model predictive control method is a modern control method applied to various technological processes [1–4]. In this approach, one constructs the prediction of the plant's behavior in the model predictive controller (MPC), which, according to the prediction of the model's input and output at finite horizons, produces a control signal for each discrete-time clock cycle. Examples of adaptive control with a predictive model are also known; in particular, it has been used to control microaerobic processes [5], for climate control [6], and so on. Sometimes adaptation of control with a predictive model is combined with methods of robust control [7] and other approaches.

This approach to control with the input and output predictions can also be used to control a complex plant such as plasma in a tokamak's magnetic field [8, 9]. In particular, in [10] a system with prediction is used in the experiment with the COMPASS-D tokamak (England) to control the vertical position of the plasma, in [11] a scalar plasma velocity stabilization system in the ITER tokamak (France) reactor is developed and simulated relative to zero with CPM, in [12] the model predictive control approach is used to solve the multivariable task of controlling the position, current and plasma shape in ITER [13] with the DINA code (State Research Center of Russian Federation Ttoitsk Institute for Innovation & Fusion Research), also for controlling the safety factor profile in ITER with variable constraints in [14] on the RAPTOR code. In all these cases of plasma control with MPC, a stationary model was used to predict the plant's behavior.

In modern tokamaks with a vertically elongated cross-section [9], the problem of stabilizing the unstable vertical position of the plasma is highly relevant and requires the development of new methods for solving it. Therefore, in this work we pose and solve the problem of controlling the unstable vertical position of the plasma for a dynamic model of a tokamak with an unknown variable parameter of the plasma by means of an adaptive MPC [15]. The plasma is stabilized by the horizontal magnetic field of the control coil, and the voltage on the coil is controlled with a multiphase controllable thyristor rectifier. To be definite, the numerical values of the plant's model parameters were chosen for a given operation mode of the T-15 tokamak currently constructed at the Kurchatov Institute [16].

2. STATEMENT OF THE PROBLEM

Taking into account that the plasma model in the tokamak can vary with time, in this work we solve the control problem with a predictive model with variable parameters. In addition, values of variable parameters may not always be known, which requires their evaluation at the rate of observation. This leads to the formulation of the problem of constructing an adaptive controller able to evaluate parameters on the input and output signals of the plant and apply them in the predictive model.

The plasma column is elongated vertically to increase the plasma pressure for the same toroidal magnetic field [17]. A horizontal magnetic field B_R is used to elongate the cross-section of the column. In case of a perturbation such as, for example, an upward displacement of the plasma, the symmetry of the distribution of currents and magnetic fields is violated, and the current in the upper part relative to the equatorial plane becomes larger than in the lower part. Then, according to Ampere's law, the force $F = [I_p \times B_r]$ [18], where I_p is the plasma current acting on the upper part of the plasma will be greater than the force acting on the lower part, so the resultant force, directed upwards, will move the plasma up the Z axis. In the absence of a control action, the displacement of the plasma is irreversible since the resultant force directed upwards will increase [15, 19, 20] (Fig. 1).

The controlled plant model is represented as a series connection of the nonstationary plasma model (1), the control coil model (2), and the thyristor rectifier model (3):

$$T_p(t)\frac{dZ(t)}{dt} - Z(t) = K_p(t)I(t), \tag{1}$$

$$T_c\frac{dI(t)}{dt} + I(t) = K_c(t)U(t), \tag{2}$$

$$T_a\frac{dU(t)}{dt} + U(t) = K_a(t)V(t), \tag{3}$$

where Z is the vertical plasma displacement, I, U are the current and voltage in the control coil, V is the rectifier input, K_p changes in the range [1.78; 7.61] cm/kA, $T_p \in [20.8;\ 43.4]$ ms, $K_c = 11.11$ Ohm^{-1}, $T_c = 46.7$ ms, $K_a = 2000$, $T_a = 3.3$ ms [15, 20]. In the general case, the parameters of the plasma model vary with time, which is reflected in the Eq. (1) as variables of the

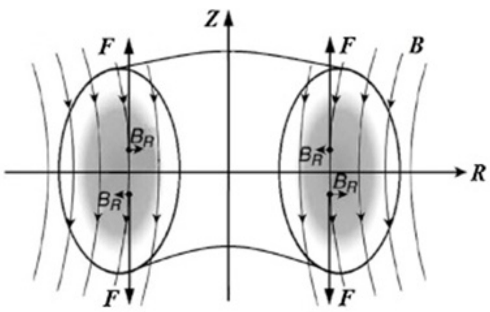

Fig. 1. The physics of vertical plasma instability arising in a tokamak.

gain coefficient K_p and parameter T_p. In this case, all states of the plant model are available for measurement.

The model (1)–(3) in the state space has the form

$$\dot{x}(t) = A_c(t)x(t) + B_c u(t), \quad y(t) = C_c x(t) \tag{4}$$

where t denotes continuous time and

$$A_c(t) = \begin{bmatrix} \dfrac{1}{T_p(t)} & \dfrac{K_p(t)}{T_p(t)} & 0 \\ 0 & -\dfrac{1}{T_c} & \dfrac{K_c}{T_c} \\ 0 & 0 & -\dfrac{1}{T_a} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 & \dfrac{K_a}{T_a} \end{bmatrix}, \quad C_c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} Z \\ I \\ U \end{bmatrix}.$$

The control objective for the unstable plant model (1)–(3) is to suppress instability and track the output Z for a reference action r. In this case, the problem statement consists of two stages. First, the problem is to stabilize an unstable plant with known variable parameters $K_p(t)$, $T_p(t)$, i.e.,

$$|Z(t) - r(t)| < \varepsilon, \quad t \in [t_0,\ t_1],$$

where ε specifies the stabilization accuracy, for example for a T-15 tokamak $\varepsilon = 1$ cm on the finite control interval $[t_0,\ t_1]$. Then the task becomes more complicated: the controlled plant has an unknown variable parameter $a(t) = K_p(t)/T_p(t)$ (with a known variable parameter $b(t) = 1/K_p(t)$), which should be evaluated at the rate of observation in order to use the estimate in the control algorithm. The accuracy level ε should be maintained.

3. MODEL PREDICTIVE CONTROL

To design the MPC with variable parameters, we assume quasi-stationarity of the controlled plant model, since at each time moment parameters of the plant vary slowly enough compared to the processes that occur in the control system. By a continuous system of Eqs. (4) of the controlled plant, we construct a discrete implementation of the model $(A_m(t), B_m, C_m)$ with sampling time interval T_0 by a zero order extrapolation in MATLAB [21]. To construct the MPC, we move from the discrete implementation $(A_m(t), B_m, C_m)$ of the discrete-time model $k = 0, 1, 2, \ldots$, with sampling period T_0, discrete-time state vector $x_m(k)$, and time-variable matrix $A_m(k)$

$$x_m(k+1) = A_m(k)x_m(k) + B_m u(k), \quad y(k) = C_m x_m(k),$$

to the extended discrete implementation $(A(k),\ B,\ C)$ of the system model [4] in order to introduce a discrete integrating unit into the feedback that would be built into the predictive model at its output. For this purpose, we introduce increments of the state vector and the input action

$$\Delta x_m(k) = x_m(k) - x_m(k-1), \quad \Delta u(k) = u(k) - u(k-1)$$

and the difference equation of the extended model is represented in the new variables x, y and with a new input, namely the increment at each time step $\Delta u(k)$, in the form

$$\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_m(k) & o_m^{\intercal} \\ C_m A_m(k) & 1 \end{bmatrix}}^{A(k)} \overbrace{\begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix} B_m \\ C_m \end{bmatrix}}^{B} \Delta u(k),$$

$$y(k) = \overbrace{\begin{bmatrix} o_m & 1 \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}.$$

The predicting controller at each time step finds a set of increments of input actions, N_c in total, that minimize the functional

$$J = (R_s - Y)^{\intercal} (R_s - Y) + \Delta U^{\intercal} \bar{R} \Delta U, \tag{5}$$

where $R_s = \overbrace{[1\ 1 \ldots 1]}^{N_c}{}^{\intercal}\, r(k_i) = \overline{R_s} r(k_i)$, k_i is the instant discrete time moment, $r(k_i)$ is the desired output signal's trajectory, $\overline{R}$ is the weight matrix on the input interval of length N_c ticks, Y is the vector of output values on the output horizon of N_p ticks, ΔU is an array of increments of the input signal of length N_c. The index i is introduced into the variable k in order to take into account the predictions of future impacts in the controller's predictive model relative to the current moment k_i.

The predicted values of the input and output horizons are calculated according to the extended implementation $(A(k), B, C)$ one by one, successively. Introducing the notation $x(k_i + m \mid k_i)$ for the predicted value on m steps forward at the (k_i)th tick, we can write the vectors of the resulting states, output and input signals Y and ΔU, as

$$x(k_i + m \mid k_i) = A(k_i)^m x(k_i) + A(k_i)^{m-1} B \Delta u(k_i)$$
$$+ A(k_i)^{m-2} B \Delta u(k_i + 1) + \ldots + B \Delta u(k_i + m - 1),$$

$$Y = [y(k_i + 1 \mid k_i) \quad y(k_i + 2 \mid k_i) \quad y(k_i + 3 \mid k_i) \quad \ldots \quad y(k_i + N_p \mid k_i)\ ]^{\intercal}, \quad \dim Y = N_p,$$
$$\Delta U = [\Delta u(k_i) \quad \Delta u(k_i + 1) \quad \Delta u(k_i + 2) \quad \ldots \quad \Delta u(k_i + N_c - 1)\ ]^{\intercal}, \quad \dim \Delta U = N_c.$$

Then the array of signal values at the output horizon is represented in the form

$$Y = F(k_i) x(k_i) + \Phi(k_i) \Delta U,$$

where the matrices equal

$$F(k_i) = \begin{bmatrix} CA(k_i) \\ CA(k_i)^2 \\ CA(k_i)^3 \\ \vdots \\ CA(k_i)^{N_p} \end{bmatrix},$$

$$\Phi(k_i) = \begin{bmatrix} CB & 0 & \ldots & 0 \\ CA(k_i)B & CB & \ldots & 0 \\ CA(k_i)^2 B & CA(k_i)B & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ CA(k_i)^{N_p-1} B & CA(k_i)^{N_p-2} B & \ldots & CA(k_i)^{N_p-N_c} B \end{bmatrix}.$$

The expressions above show that the matrix $F(k_i)$ is determined by the matrices of the plant model A, C and also by the output horizon value N_p, and the matrix $\Phi(k_i)$ is determined by three plant matrices A, B, C and the lengths of input and output horizons N_p, N_c. The optimal set of increments for the inputs is

$$\Delta U = \left(\Phi(k_i)^{\intercal} \Phi(k_i) + \bar{R}\right)^{-1} \Phi(k_i)^{\intercal} \left(R_s - F(k_i) x(k_i)\right),$$

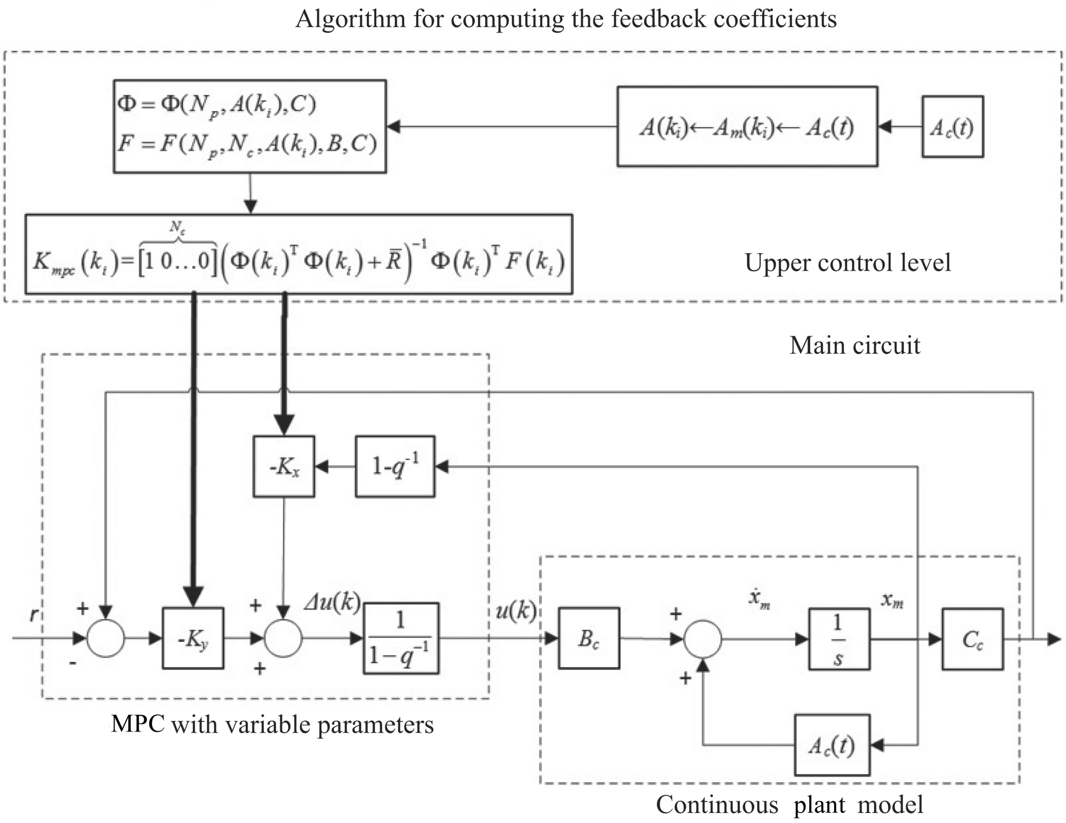

Fig. 2. Structural flowchart of the hybrid control system with non-stationary plant model in continuous time and MPC with variable parameters: bold arrows indicate parametric influence on the MPC.

where $(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}$ is the Hessian of the extended system, which can be found from the necessary condition for the existence of an extremum of the functional (5): $\partial J/\partial \Delta U = 0$. In the control law, only the first increment of the input is applied at time k_i:

$$\Delta u\,(k_i) = \overbrace{[1\ 0 \ldots 0]}^{N_c}\,(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}(\Phi(k_i)^{\mathsf{T}}R_s - \Phi(k_i)^{\mathsf{T}}F(k_i)x\,(k_i))$$
$$= K_y(k_i)r\,(k_i) - K_{mpc}(k_i)x\,(k_i)\,,$$

where the number

$$K_y(k_i) = \underbrace{\begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}}_{N_c}(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}\Phi(k_i)^{\mathsf{T}}\overline{R_s} \tag{6}$$

is the first element of the vector $(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}\Phi(k_i)^{\mathsf{T}}\overline{R_s}$, and the row vector

$$K_{mpc} = \underbrace{\begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}}_{N_c}(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}\Phi(k_i)^{\mathsf{T}}F(k_i) \tag{7}$$

is the first row of the matrix $(\Phi(k_i)^{\mathsf{T}}\Phi(k_i) + \bar{R})^{-1}\Phi(k_i)^{\mathsf{T}}F(k_i)$ (the receding horizon principle). The discrete equation in the state space of the closed system with prediction becomes

$$x\,(k+1) = (A(k) - BK_{mpc})\,x\,(k) + BK_yr\,(k)\,. \tag{8}$$

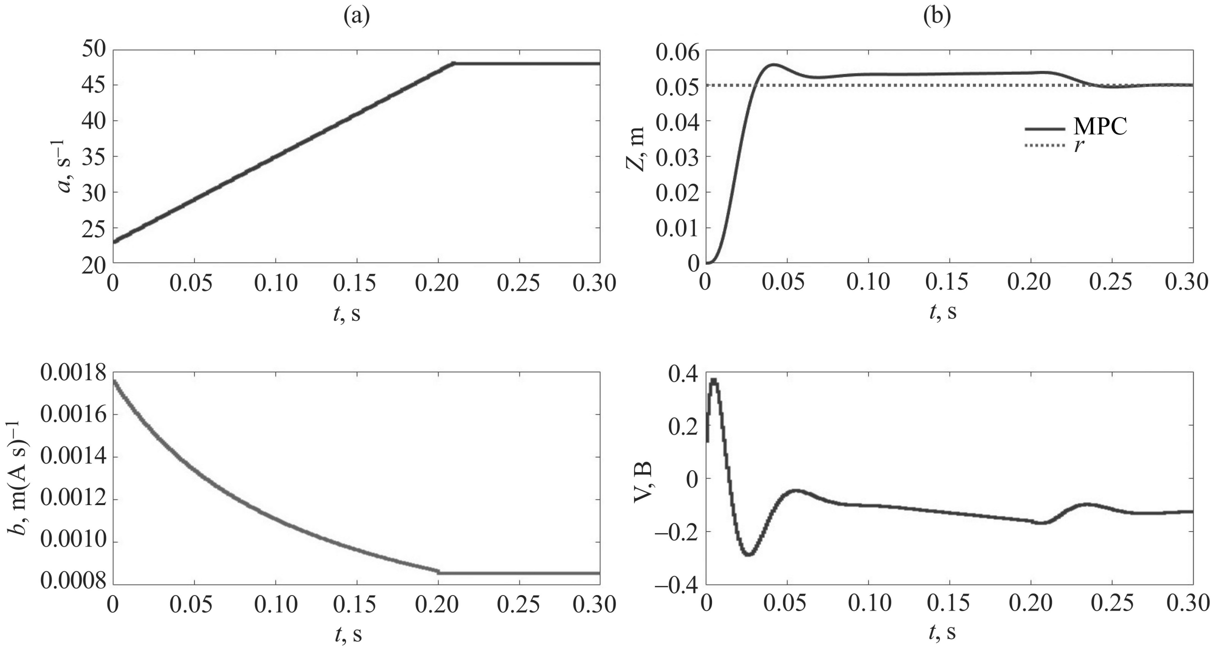

Fig. 3. Operation of the quasi-stationary control system: (a) changes of the coefficients $a(t)$ and $b(t)$ in time, (b) transient processes in the control system with non-stationary MPC.

Since the vector x consists of two components $x(k) = [\ \Delta x_m^{\mathsf{T}}(k) \quad y(k)\ ]^{\mathsf{T}}$, the row vector $K_{mpc}(k_i)$ can also be represented as two components $K_{mpc}(k_i) = [\ K_x(k_i) \quad K_y(k_i)\ ]$, where the row vector $K_x(k_i)$ is multiplied by the vector $\Delta x_m(k)$, and the number $K_y(k_i)$ is multiplied by the scalar output of the plant $y(k)$. If we set the durations of the horizons N_c and N_p, then if we know the matrices of the extended plant model (A, B, C), we can calculate feedback coefficients $K_x(K_i)$ and $K_y(K_i)$ in (8), which in turn can be used to construct a discrete MPC. In this system, the control horizon is assumed to be 20 cycles, and the observation horizon is 40 cycles with a sampling time of 1 ms. Below on the structural diagrams of Figs. 2 and 4 the symbol q^{-1} denotes the shift operator by one clock cycle back. We can see that the output of the MPC includes a discrete integrating unit with operator $1/(1 - q^{-1})$, which is a consequence of the principle of constructing an extended model of the controlled plant.

The resulting variable values are included in the MPC algorithm for a non-stationary plant (Fig. 2). The MPC has a feedback loop with coordinate influences on the controlled plant [22], and also includes parametric program actions from the direct circuit with an algorithm for computing feedback coefficients. In this case, the system has a hierarchical structure [23]. The lower level of control consists of the main loop, and the upper level influences the lower level by computing the optimal prediction and the optimal influence at every clock cycle.

Transient processes under the influence of the Heaviside function on the input of the closed system (8) with nonstationary MPC under variable parameters of the plant model (Fig. 3a) are shown in Fig. 3b.

The hybrid control system with MPC was simulated in the Simulink graphical simulation environment. To model a system with variable parameters and compute the feedback coefficients at control rate, we developed a Simulink unit that takes variable parameters of the plant model as input signals and outputs the feedback coefficients K_x and K_y as output signals. This approach made it possible to provide a simulation of the quasi-stationary system and its controller with acceptable accuracy and clarity.

4. ESTIMATING THE VARIABLE PARAMETER

When controlling a real plant, the value of one of the parameters of the plasma model may be unknown. In order to build the MPC, we have to estimate the model coefficient, which varies in time. Then we can substitute the resulting value of the coefficient into the predictive model and obtain the control of the plant model with a variable parameter. Suppose that the law of how $a(t)$ changes is unknown. Let us construct an algorithm for estimating the parameter $a(t)$ using the gradient descent method for the state observer [25] with known variable parameter $b(t)$. The observer equation for the plasma model (1) is

$$\dot{\widehat{Z}}(t) = \widehat{a}(t)\widehat{Z}(t) + b(t)I(t) + \gamma\left[Z(t) - \widehat{Z}(t)\right], \tag{9}$$

where $\widehat{Z}$ is the state estimate, $\widehat{a}$ is the estimate of the unknown parameter a, γ is a constant positive coefficient. From the Eq. (9) we can find an estimate of the output state $\widehat{Z}$, which corresponds to (1) for $\widehat{Z} \to Z$. This makes it possible to write down the algorithm of automatic estimation for the parameter $\widehat{a}$ in the form

$$\dot{\widehat{a}}(t) = -\lambda\frac{\partial Q(t)}{\partial\widehat{a}},$$

where we have introduced a quadratic quality criterion for the observer's approximation of the plant model $Q(t) = \left[Z(t) - \widehat{Z}(t)\right]^2$, λ is a constant positive parameter. After taking the partial derivative, we obtain

$$\dot{\widehat{a}}(t) = 2\lambda\left[Z(t) - \widehat{Z}(t)\right]\beta(t), \tag{10}$$

where $\beta = \partial\widehat{Z}/\partial\widehat{a}$ is the sensitivity function of the observer output $\widehat{Z}$ to the estimate of parameter $\widehat{a}$. Differentiating the observer's Eq. (9) with respect to the parameter estimate $\widehat{a}$ and swapping the derivatives, we arrive at the differential equation for the sensitivity function

$$\dot{\beta}(t) = \widehat{Z}(t) + [\widehat{a}(t) - \gamma]\beta(t). \tag{11}$$

Solution of the system of three differential Eqs. (9)–(11) yields an estimate of the parameter $\widehat{a}$.

Passing from Z and a to deviations $\delta_Z = Z - \widehat{Z}$ and $\delta_a = a - \widehat{a}$, we obtain a system of gradient descent equations in deviations to investigate the convergence of the method:

$$\begin{aligned}\dot{\delta}_Z &= Z\delta_a - \delta_a\delta_Z + (a-\gamma)\delta_Z,\\ \dot{\delta}_a &= -2\lambda\delta_Z\beta,\\ \dot{\beta} &= Z - \delta_Z + (a-\gamma)\beta - \beta\delta_a.\end{aligned} \tag{12}$$

In this case, for $Z = 0.05$ m and $a = 48\ \mathrm{s}^{-1}$, $\gamma = 500\ \mathrm{s}^{-1}$ the equilibrium point for the sensitivity function β will be equal to $\beta^* = z/(\gamma - a) = 1.1 \times 10^{-4}$ m $\times$ s. The resulting nonlinear system of equations with respect to deviations in Z, a and β was investigated for convergence by mathematical modeling near equilibrium points. As a result of point-by-point scanning of the trajectories of the solution of the equation in the three-dimensional domain

$$S = \left\{(\delta_Z, \delta_a, \beta)\,|\,0.1\ \mathrm{m} \leqslant \delta_Z \leqslant 0.1\ \mathrm{m},\ 0.01\ \mathrm{s}^{-1} \leqslant \delta_a \leqslant 0.01\ \mathrm{s}^{-1},\ 0.1\ \mathrm{m}\times\mathrm{s} \leqslant \beta \leqslant 0.1\ \mathrm{m}\times\mathrm{s}\right\}$$

obtained with the *ode45* algorithm in MATLAB, we have found that initial conditions lying on a certain surface (Fig. 4) whose shape can be specified with variable parameters λ and γ, always come

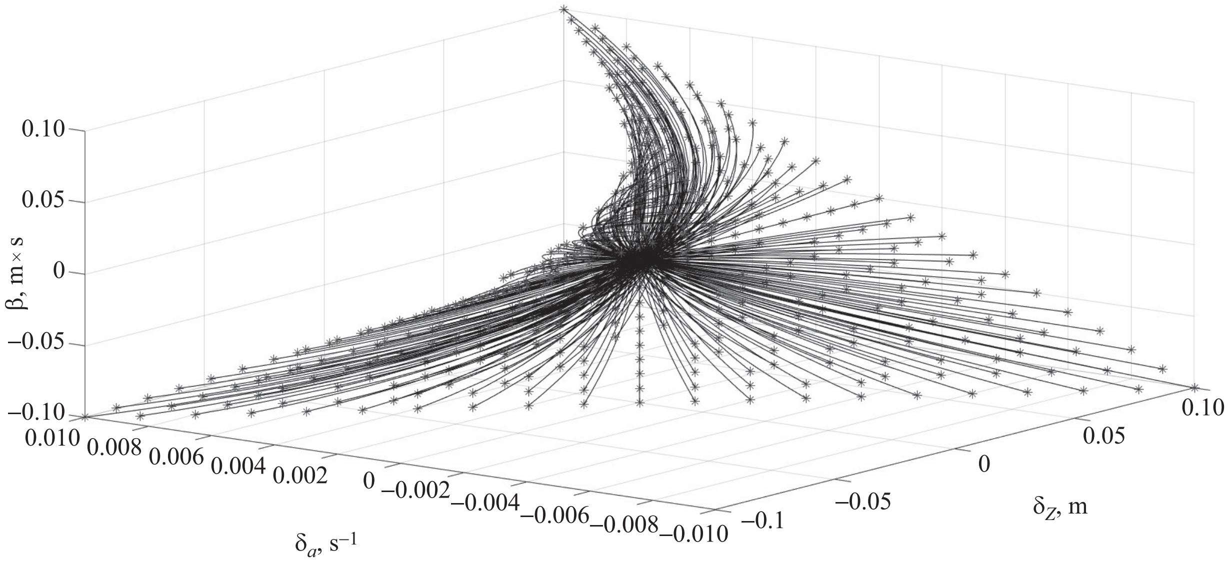

Fig. 4. The surface of the initial conditions from which trajectories converge to the equilibrium point corresponding to the exact identification of the parameter.

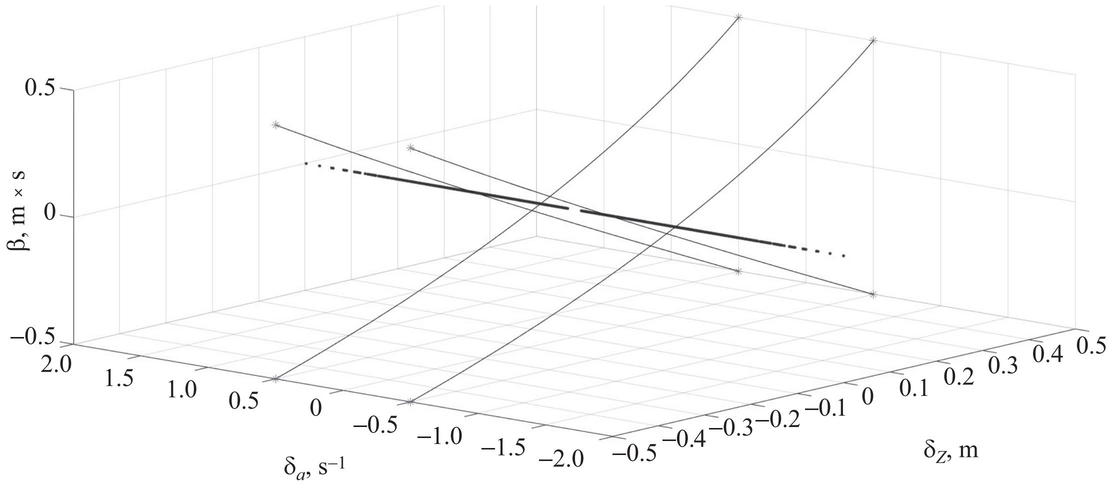

Fig. 5. A straight line to which the trajectories converge starting from initial conditions that do not lie on the surface of convergence of the trajectories to zero.

to the equilibrium point corresponding to the exact identification of the desired plant parameter. Thus, with properly chosen initial conditions and parameters λ and γ the identification error a tends to zero. The end points of trajectories originating outside a given surface converge to a straight line parallel to the axis δ_a and correspond to the error for an incorrect selection of parameters (Fig. 5). Thus, in order to obtain a trajectory going to the origin from an arbitrary point of the initial conditions, we have to choose the coefficients in such a way that the surface passes through this point. Also, analysis of the derivative of the Lyapunov function

$$V = \delta_Z^2 + \delta_a^2 + (\beta - \beta^*)^2 \tag{13}$$

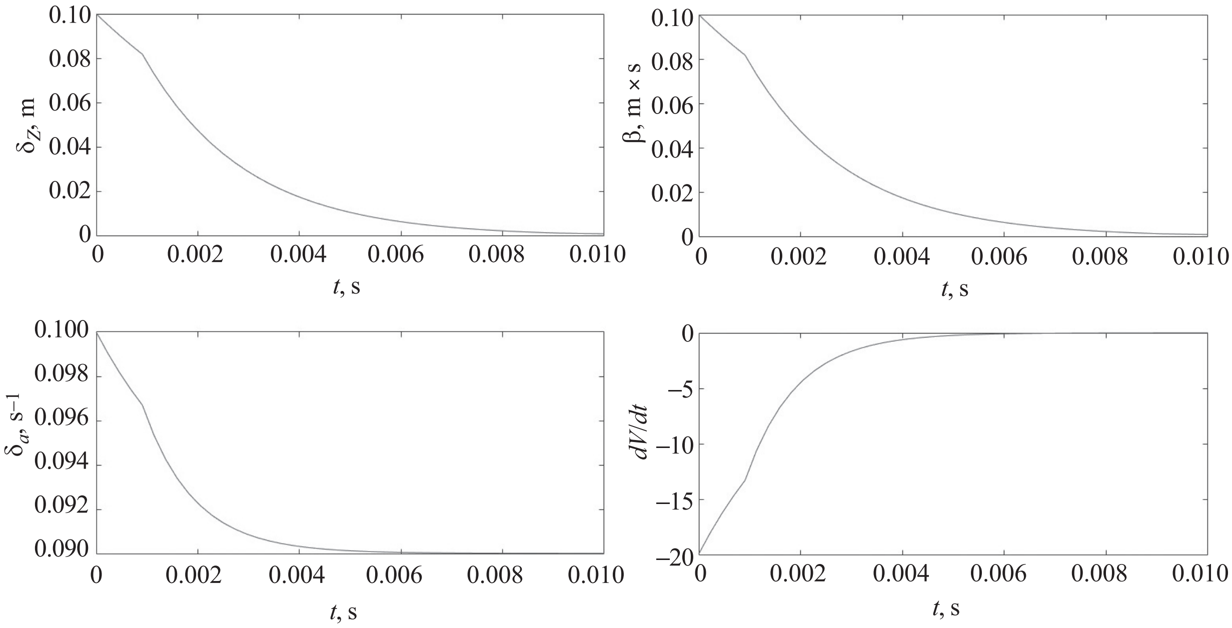

Fig. 6. The result of modeling the system in deviations with respect to three variables and the derivative of the Lyapunov function.

with respect to the system

$$\begin{aligned}\dot{V} &= 2\delta_Z\dot{\delta}_Z + 2\delta_a\dot{\delta}_a + 2\beta\dot{\beta}\\ &= 2\delta_Z\left(Z\delta_a - \delta_a\delta_Z + (a-\gamma)\delta_Z\right) + 2\delta_a(-2\lambda\delta_Z\beta) + 2\beta(Z - \delta_Z + (a-\gamma)\beta - \beta\delta_a)\end{aligned}$$

has showed that its sign is negative throughout the identification time interval (Fig. 6).

5. CONTROL SYSTEM WITH ADAPTIVE MPC

Let us construct an adaptive MPC for the control of a non-stationary plant with an unknown parameter. Estimate of the plant matrix $\widehat{A}_c(t)$ in continuous time takes the form

$$\widehat{A}_c(t) = \begin{bmatrix} \widehat{a}(t) & b(t) & 0 \\ 0 & -\dfrac{1}{T_c} & \dfrac{K_c}{T_c} \\ 0 & 0 & -\dfrac{1}{T_a} \end{bmatrix}.$$

Let us pass to an estimate of the discrete system matrix $\widehat{A}_m(t)$, again by transitioning to the expanded quasi-stationary model of the plant:

$$\overbrace{\begin{bmatrix} \Delta x_m\,(k+1) \\ y\,(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} \widehat{A}_m(k) & o_m^{\intercal} \\ C_m\widehat{A}_m(k) & 1 \end{bmatrix}}^{\widehat{A}(k)} \overbrace{\begin{bmatrix} \Delta x_m\,(k) \\ y\,(k) \end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix} B_m \\ C_m \end{bmatrix}}^{B} \Delta u\,(k),$$

$$y\,(k) = \overbrace{[o_m\ 1]}^{C} \begin{bmatrix} \Delta x_m\,(k) \\ y\,(k) \end{bmatrix}.$$

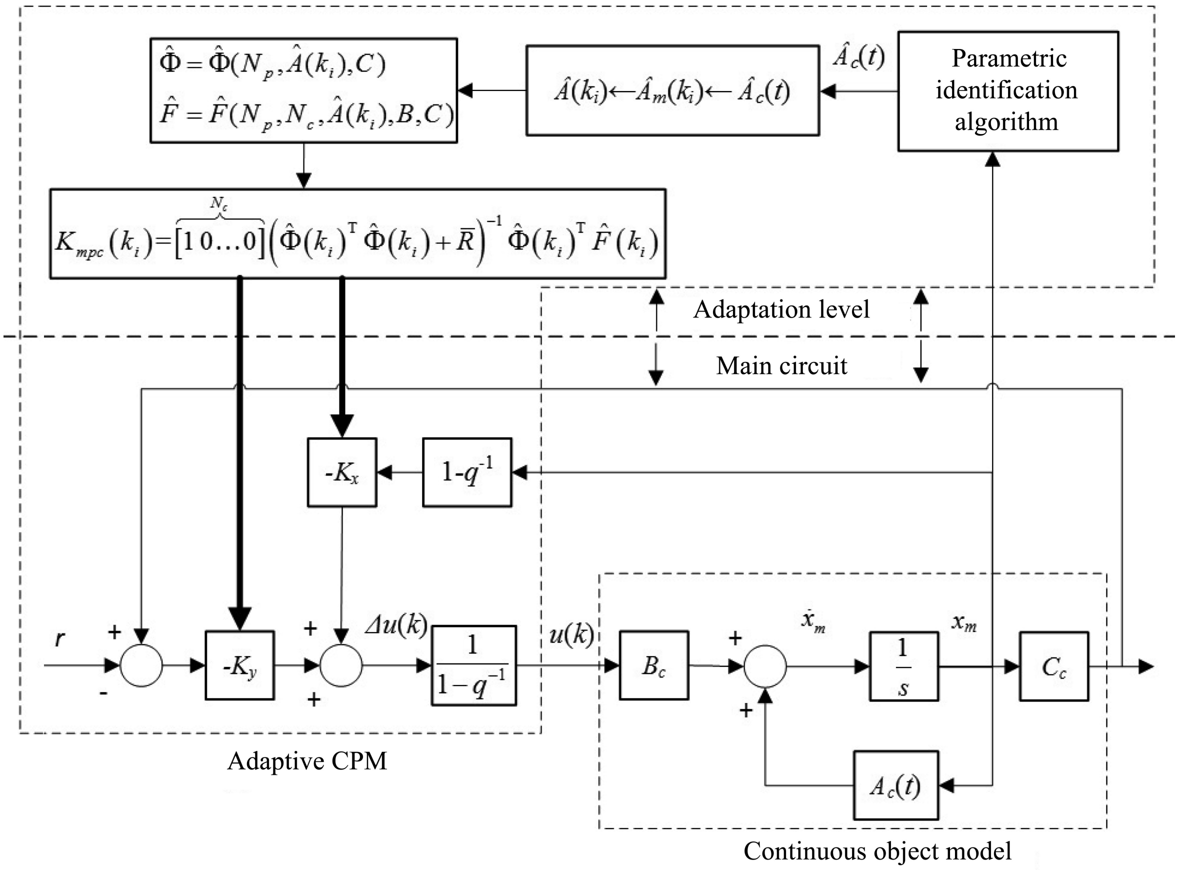

Fig. 7. Flowchart of a hybrid control system with adaptive MPC and non-stationary plant model with unknown variable parameter.

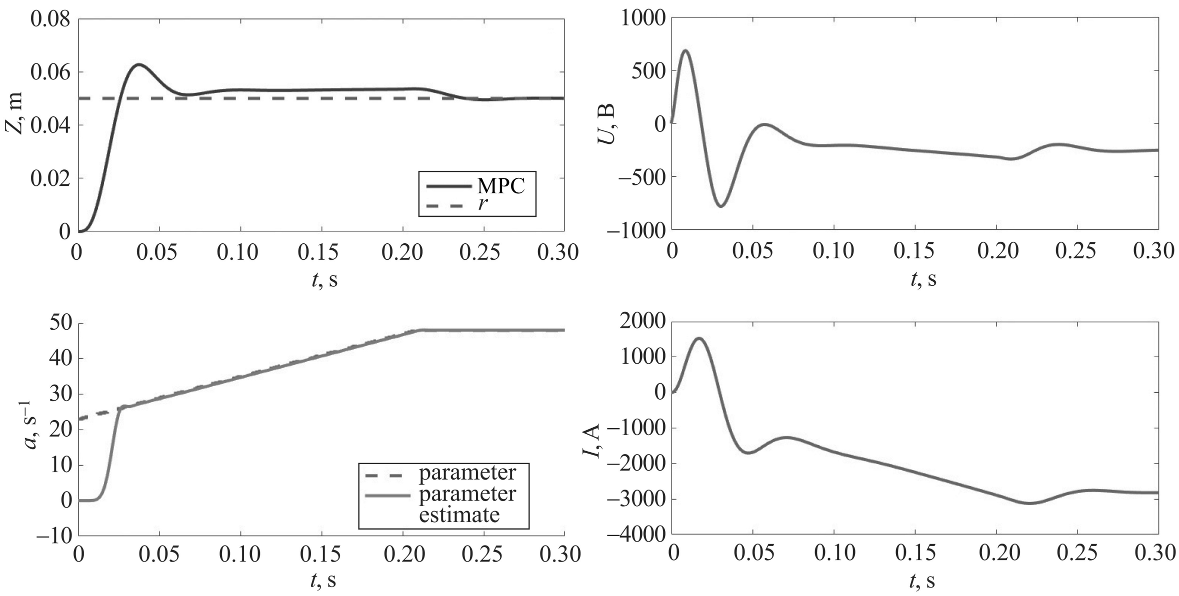

Fig. 8. Transient processes in the adaptive system with estimate of the parameter $a(t)$ for the plant model with a variable unknown parameter.

The optimal set of increments on every time step now takes the form

$$\Delta U = \left(\widehat{\Phi}(k_i)^\intercal \widehat{\Phi}(k_i) + \bar{R}\right)^{-1} \widehat{\Phi}(k_i)^\intercal \left(R_s - \widehat{F}(k_i) x\,(k_i)\right),$$

$$K_{mpc}\,(k) = \overbrace{[1\ 0 \ldots 0]}^{N_c} \left(\widehat{\Phi}(k_i)^\intercal \widehat{\Phi}\,(k_i) + \bar{R}\right)^{-1} \widehat{\Phi}(k_i)^\intercal \widehat{F}\,(k_i)\,.$$

In the adaptive case, the known matrices B and C are used in the construction of $\widehat{\Phi}(k_i)$ and $\widehat{F}(k_i)$, and instead of the unknown matrix variable $A(k_i)$ we use its estimate $\widehat{A}(k_i)$. The choice of feedback coefficients for the first element ΔU is similar to the quasistationary case with known parameters of the plant model.

In the simulation of the adaptive control system in Simulink (Fig. 7), the unit shown in Fig. 2 received as input not the variable parameter $a(t)$ but its estimate $\widehat{a}(t)$ obtained with an adaptive observer with the system of differential Eqs. (9)–(11).

A non-stationary model of the plant (1)–(3) was applied to the resulting control system with adaptive MPC. Figure 8 shows the process of estimating parameter a and transient processes in the adaptive system with a stepwise input influence.

6. CONTROLLING THE MODEL OF A PLANT WITH VARIABLE STRUCTURE

If the law according to which parameter $a(t)$ changes has alternating signs, then the plant model (1)–(3) is unstable during time intervals when parameter $a(t)$ is positive and stable when $a(t)$ is negative, i.e., the structure of the plant model changes.

When modeling the control system, the parameter $a(t)$ changed according to the sine law to test the sensitivity of the system with adaptive MPC to the sign of $a(t)$. Estimation results for the variable parameter and the simulation of the control system with adaptive MPC with a step-like input influence are shown in Fig. 9. In this case, the system with adaptive MPC remains stable when the structure of the plant model changes and tracks the reference influence with acceptable accuracy.

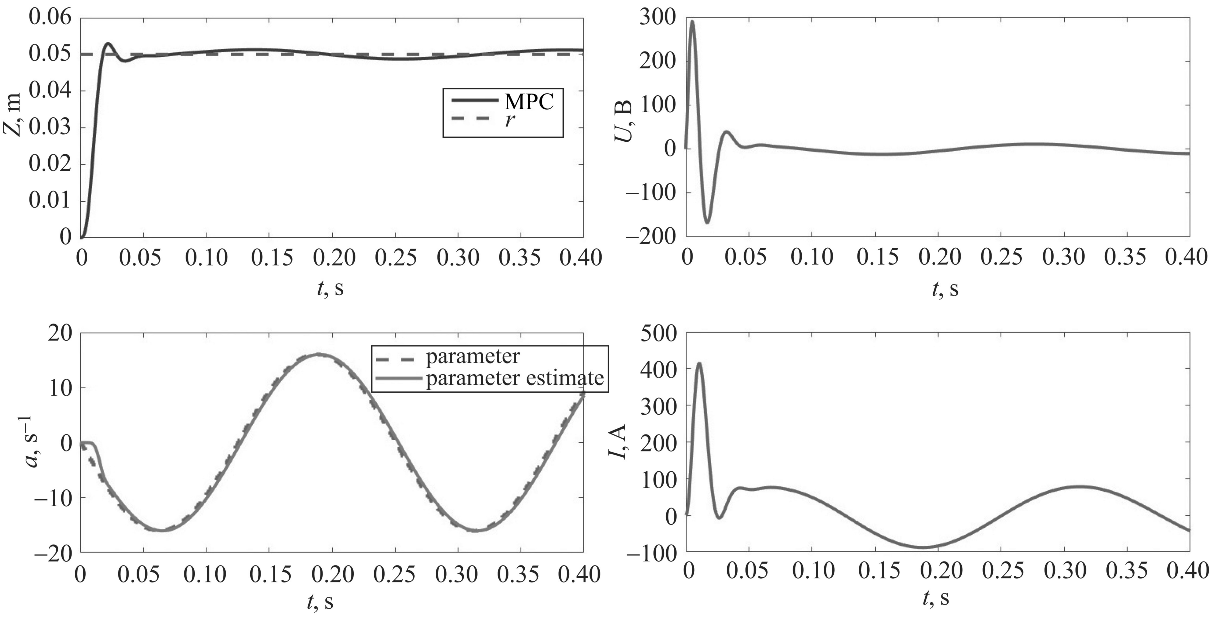

Fig. 9. Transient processes in the adaptive system with estimation of the parameter $a(t)$ with the plant model of variable structure.

7. CONCLUSION

In this work, we have developed the control method with predictive model [1–7] and applied it to the model of an unstable nonstationary plant of the third order, which represents a model of vertical plasma motion in modern tokamaks with a cross-section elongated in the vertical direction. As a specific example of the plant model, we have chosen the plasma dynamics model in the T-15 tokamak [16, 20]. Modeling in the Simulink graphical simulation environment has shown the efficiency of the proposed hybrid control system with a predictive adaptive discrete model.

We see further development of the adaptive control method with the predictive model in its applications to other problems of plasma control in tokamaks, in particular, for solving the problem of controlling the plasma current in the Globus-M tokamak (Ioffe Physicotechnical Institute, St. Petersburg) [25] while taking into account constraints on the current and voltage of the coil of the central solenoid.

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