Classical soliton theory for studying the dynamics and evolution of cylindrical shock waves in passive dispersed and active relaxation media

Sergey Belyakin[1,*], Sergey Shuteev[1,2]

[1]Department of General Physics, Physics Faculty, Lomonosov Moscow State University, Moscow, Russia.

[2]Laboratory of dynamic systems, Physics Faculty, Lomonosov Moscow State University, Moscow, Russia.

Email address: belyakin1962@mail.ru (author 1), shuteev@phys.msu.ru (author 2)

Abstract: This paper presents the dynamic model of the soliton. Based on this model, it is supposed to study the state of the shock wave and the transition of the wave to another state. The theory and experiment of cylindrical shock waves is presented in the literature [1–4].

Keywords: Nonlinear dynamic system, cylindrical shock waves, soliton, passive dispersed media, active relaxation media.

1. Introduction

Shock waves in low-temperature plasma are of interest both from scientific and applied fields. The shock wave causes a rapid increase in temperature and pressure in the discharge area. The release of energy in the discharge before the shock wave front affects its speed and other characteristics. In the experiment, it was found that the wave front has two stages in the structure. A second reflected wave is observed inside the cylinder. In this experiment, the secondary wave is formed at smaller distances from the edge of the cylinder, in the absence of a discharge, the absence of a second front [5].

2. The mathematical dynamic model of the soliton

The mathematical dynamic model of the soliton is represented by the equation (1).

$$\begin{aligned} \dot{x} &= x(1 - a_3 y - a_1 x)(1 + k\sin\omega t), \\ \dot{y} &= by(1 - a_3 x - a_2 y)(1 + k\cos\omega t), \\ \dot{z} &= xy\, schx\, schy \sin\omega t \cos\omega t. \end{aligned} \tag{1}$$

Time portraits of the system (1) are shown in Fig.(1–3), active relaxation media: a =1.0, a_1 = 0.2, a_2 = 0.1, a_3 = 1.0, b = –2.0, k = 0.045, ω = 64π.

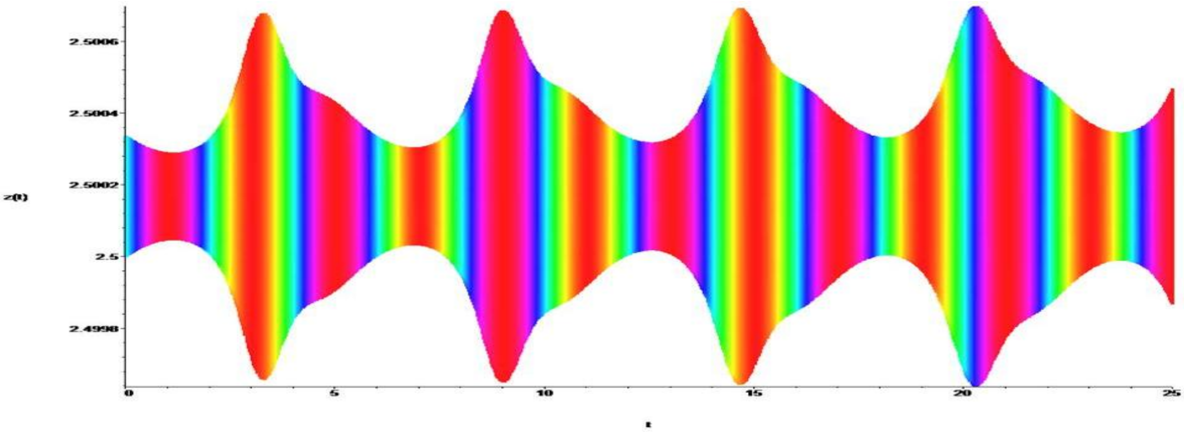

Fig.1. Time portraits of the system (1) at : x_0 = 0.4, y_0 = 0.4, z_0 = 2.5, t = (0→ 25).

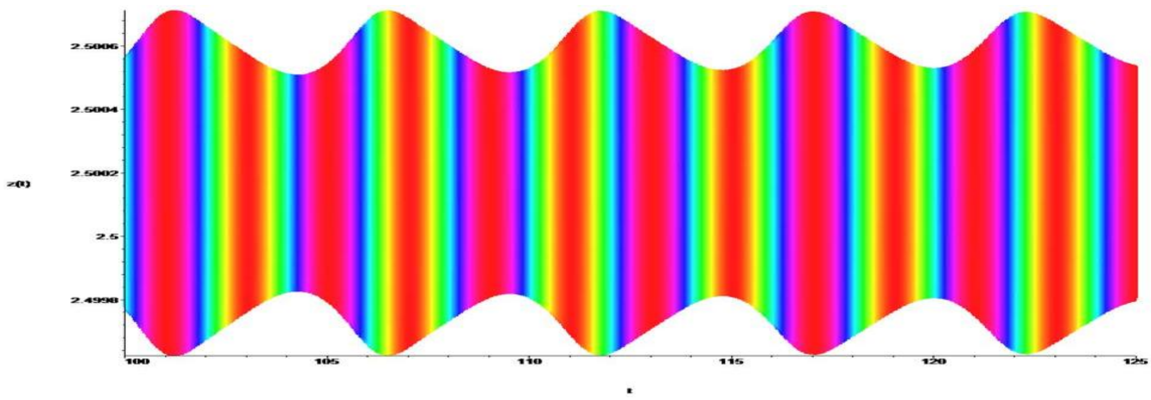

Fig.2. Time portraits of the system (1) at : x_0 = 0.4, y_0 = 0.4, z_0 = 2.5, t = (100→ 125).

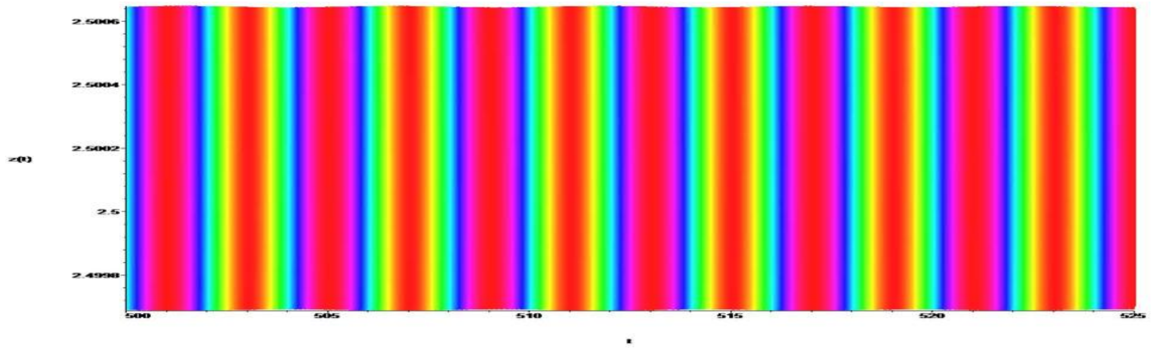

Fig.3. Time portraits of the system (1) at : x_0 = 0.4, y_0 = 0.4, z_0 = 2.5, t = (500→ 525).

3. Dynamics of convergence of a cylindrical shock wave to the 0Z axis of symmetry and reflection from its Z plane

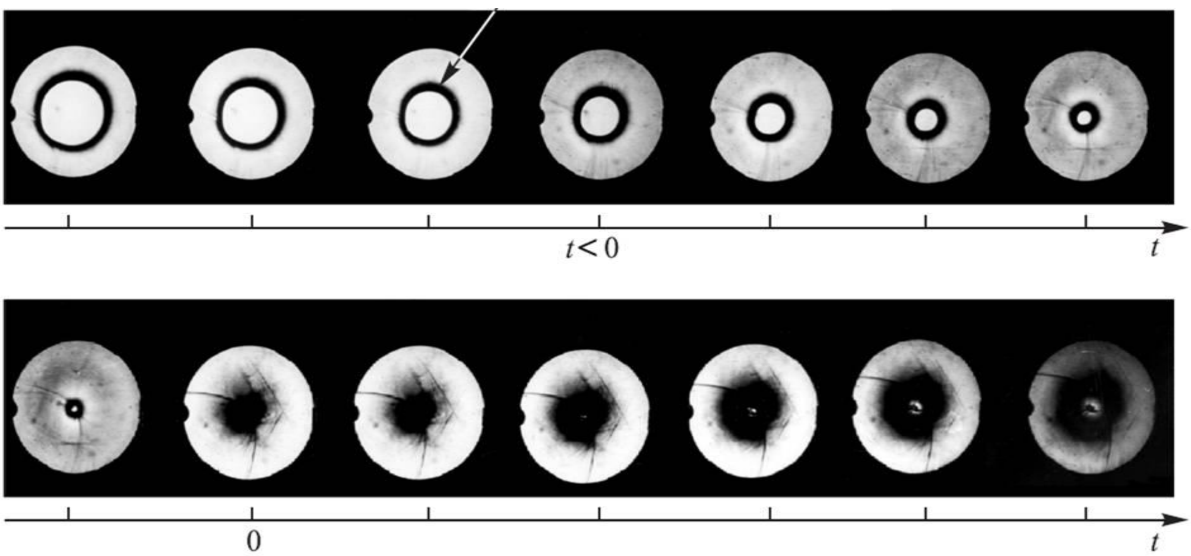

Fig.4. The process of convergence of a cylindrical shock wave to the axis of symmetry 0z and reflection from it. The moment of shock wave accumulation on the axis 0z is taken as t = 0. The

duration of the sequence of photos recording the process of convergence of the cylindrical shock wave front is equal to 2 mcc. Radius of the ring R_0 = 0.05 m.

The dynamics of convergence of a cylindrical shock wave to its axis of symmetry was studied (0z). This was achieved by registering the shadow pattern of the shock wave convergence perpendicular to the direction axis [6]. Preliminary experiments have shown that the effect of cumulation of a converging shock wave is manifested in a relatively small region (10–20 mm) in the vicinity of the axis (0z). Therefore, in order to detect a converging shock wave, special attention is paid to the spatiotemporal resolution of the wave in this region. The fig.4 shows a sequence of photos recording the process of convergence of the front of a cylindrical shock wave. The process of convergence of the shock wave front depends on the initial conditions [7].

Time portraits of the system (1) are shown in Fig.(5,6), passive dispersed media: a =–1.0, a_1 = 0.2, a_2 = 0.1, a_3 = 1.0, b = 2.0, k = 0.045, ω = 64π.

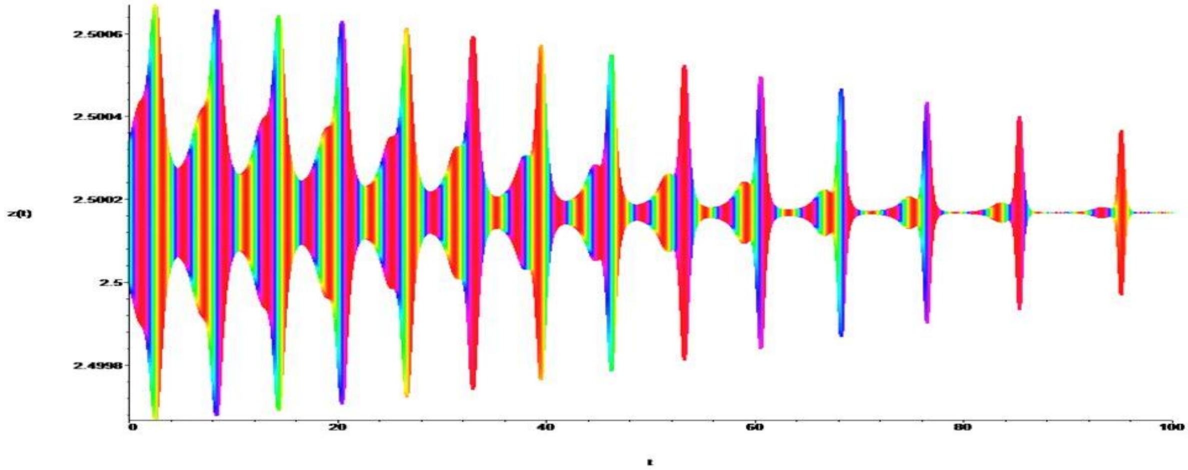

Fig.5. Time portraits of the system (1) at : x_0 = 0.4, y_0 = 0.4, z_0 = 2.5, t = (1→ 100).

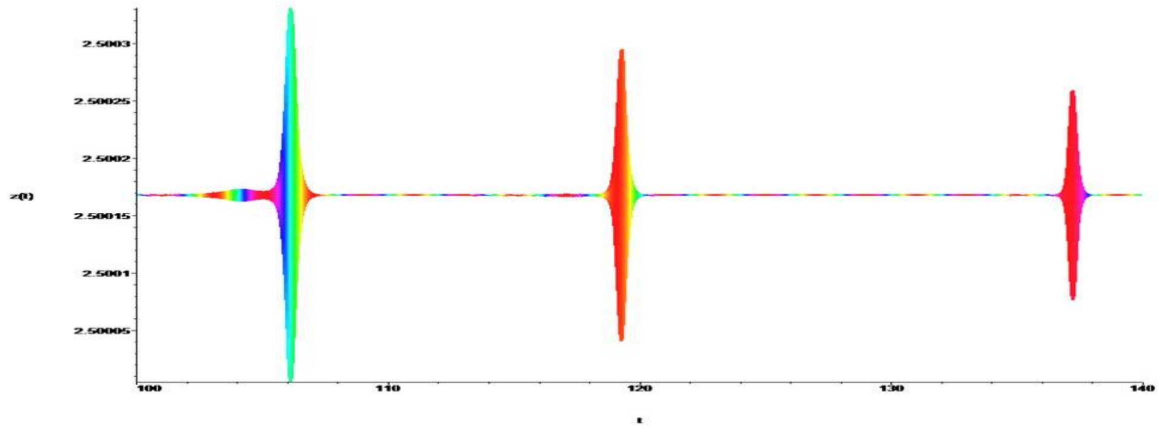

Fig.6. Time portraits of the system (1) at: x_0 = 0.4, y_0 = 0.4, z_0 = 2.5, t = (100→ 140).

4. Result

The mathematical dynamic model of the soliton is represented by equation (1) for a =1.0, a_1 = 0.2, a_2 = 0.1, a_3 = 1.0, b = –2.0, k = 0.045, ω = 64π and describes the active relaxation state. At t = (0→ 250), a cylindrical shock wave forms the leading edge of the wave (fig. 1,2). At t =

(300→ 500), the cylindrical shock wave is formed into a classical frontal wave (fig. 3). The mathematical dynamic model of the soliton is represented by equation (1) for $a = -1.0$, $a_1 = 0.2$, $a_2 = 0.1$, $a_3 = 1.0$, $b = 2.0$, $k = 0.045$, $\omega = 64\pi$ and describes the passive dispersed state. At t = (0→ 100), a cylindrical shock wave forms the back front of the wave (fig. 5). At t = (100→ 150), the front of the cylindrical shock wave contracts, the amplitude decreases, and the period increases (fig. 6).

Summary:

A modified nonlinear model of cylindrical shock waves is developed based on the classical model of a twisting spiral and two coupled classical solitons. This system of equations (1) can be useful for studying and predicting the behavior of cylindrical shock waves, processes in a homogeneous gas medium, dispersion and active medium (human atrial tissue) [8–15].

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