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SATELLITE NAVIGATION. RAW DATA PROCESSING FOR GEOPHYSICAL APPLICATIONS

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Abstract. Satellite navigation systems are widely used in geophysical applications for precise trajectory determination of a vehicle–carrier of geophysical equipment. In particular, in airborne gravimetry it is necessary to determine the velocity and acceleration of the vehicle in addition to the position determination. Mathematical models and algorithms for the solution of these problems are described. The source data consists of differential Doppler and carrier phase GPS observations.

Introduction

This paper is based on research done at the Laboratory of Control and Navigation of the Department of Mathematics and Mechanics of Lomonosov Moscow State University in close collaboration with several Russian specialized Scientific and Research Institutes.

The stimulus for this research was developing software for several Russian airborne gravimetry systems, such as those manufactured at the Moscow Institute of Electromechanics and Automatics, VNII Geophysics and Aerogeophysics, and Joint Stock Company “Gravimetric Technologies” (GT) [1]. The laboratory has cooperated with the GT company for several years. The GT-produced airborne gravimetry system GT1A (Russian name—MAG-1) was employed in several big gravimetric surveys done in Russia and abroad. The laboratory has developed post-processing software for GT1A and for the Graviton-M airborne system manufactured at the Aerogeophysics company. An essential part of this software deals with satellite navigation.

In this paper, we center on methodical issues and discuss basic approaches to satellite navigation. We shall limit our discussion to applications of Global Positioning Systems (GPS).

The main aim of processing GPS observations is to determine the position of a vehicle. This can be done with the code or carrier phase observations obtained in the standard or differential mode of GPS.

Commercial GPS software is focused on this problem [9, 10]. However, in some geophysical applications, for example, in airborne gravimetry, GPS velocity and acceleration are also required. These have to be obtained by direct processing of the raw GPS observations [4, 6, 7].

On GPS Velocity and Acceleration

Let us explain the necessity of GPS velocity and acceleration in airborne gravimetry. Consider the basic gravimetric equation [2]

$$\ddot{h} = \left(\frac{V_E^2}{R_E} + \frac{V_N^2}{R_N} + 2uV_E\cos\varphi\right) - \gamma_0 - \delta\gamma + \delta g_3 + f_3.$$

Here h is the flight altitude; the parentheses group terms called the Eötvös corrections (these corrections are the transfer and Coriolis accelerations caused by the motion of the vehicle around the Earth ellipsoid); V_E and V_N are the eastern and northern components of the vehicle velocity; R_E and R_N are the radii of curvature of the first vertical and the meridional section; u is the Earth angular velocity; φ is the geographical latitude; γ_0 is the normal gravity on the Earth surface specified by the Helmert formula

$$\gamma_0 = 9.78030(1 + 0.005302\sin\varphi^2 - 0.000007\sin^2 2\varphi) - 0.00014; \tag{1}$$

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$\delta\gamma = -2\omega_0^2 h$ is the altitude correction of normal gravity; $\omega_0 \sim 1.24 \cdot 10^{-3}$ is the Schuler frequency; f_3 is the projection of the external specific force acting on the proof mass onto the geographical vertical; δg_3 is the sought-for gravity anomaly.

Three approaches can be formulated for this problem (see [2]).

(1) The first approach assumes double integration of the gravimeter measurements and comparing the results with GPS altitude in order to extract the gravity anomaly δg from the difference of the data.
(2) The second approach, on the contrary, assumes double differentiation of the GPS altitude and comparing the results with the gravimeter readings.
(3) The third "compromise" variant assumes single differentiation and single integration, respectively, of the GPS altitude and the gravimeter readings.

The last two approaches require GPS-derived velocity and acceleration.

Obviously, velocity and acceleration can be determined by direct numerical differentiation of the GPS position. This way involves implicit differentiation of the raw GPS observations—the code pseudoranges or the carrier phases. We prefer to determine velocity and acceleration by direct processing of the raw GPS observations.

Both the Doppler and/or the carrier phase observations can be used to get GPS velocity and acceleration. The carrier phases are potentially more precise. So in practice the Doppler observations are seldom used. However, a GPS Doppler solution can be used as an initial approximation for a carrier phase solution [4, 6, 7].

Velocity Determination of a Navigation Satellite

To determine velocity and acceleration one needs to know the motion of the GPS satellites, since they enter into the solution.

Let us describe an algorithm determining the vector velocity of a GPS satellite using standard ephemeris data. These data are transmitted by the satellite and are used first of all to determine the satellites location. See details of the corresponding algorithm, e.g., in [5].

Initial ephemeris data are:

(1) the time from the ephemeris reference epoch t_{em}. For this epoch, the satellite's coordinates and velocities are computed;
(2) the reference ephemeris time T_{oe}; mean motion difference from the computed value Δn; the mean anomaly at reference time M_0; the orbit eccentricity e; the square root of the semi-major axis $\sqrt{a}$; the amplitude of the cosine and sine harmonic correction terms to the orbit radius C_{rc} and C_{rs}; the amplitude of the cosine and sine harmonic correction terms to the angle of inclination C_{ic} and C_{is}; the amplitude of the cosine and sine harmonic correction terms to the argument of the latitude C_{uc} and C_{us}; the longitude of the ascending note of the orbit plane at the weekly epoch Ω_0; the argument of the perigee ω; the inclination angle at the reference time i_0; the rate of the right ascension $\dot{\Omega}_0$; the rate of the inclination angle $\dot{i}_0$.

Determined values:

(1) $R_\eta^{\mathrm{sat}} = (R_{\eta 1}^{\mathrm{sat}}, R_{\eta 2}^{\mathrm{sat}}, R_{\eta 3}^{\mathrm{sat}})^T$—Earth Centered Earth Fixed (ECEF) satellite coordinates to the t_{em} reference epoch;
(2) $V_\eta^{\mathrm{sat}} = (V_{\eta 1}^{\mathrm{sat}}, V_{\eta 2}^{\mathrm{sat}}, V_{\eta 3}^{\mathrm{sat}})^T$—ECEF satellite components of the relative velocity to the t_{em} reference epoch t_{em}.

The following sequence of calculations is used (the standard algorithm of the satellite location is described for the determination of the ECEF coordinates and velocities of the GPS satellite [5] and supplemented with some formulas for velocity determination).

(1) The time from the ephemeris reference epoch $t^* = t_{\text{em}} - T_{\text{oe}}$. If $t^* > 302400$, then t^* is replaced by $t^* - 604800$. If $t^* < -302400$, then t^* is replaced by $t^* + 604800$. (Here $N_{\text{sec}} = 604800$ is the number of seconds in a week, $N_{\text{sec}}/2 = 302400$.)

(2) The mean anomaly M at the moment t^*:

$$M = M_0 + \left(\frac{\sqrt{\mu}}{a^{3/2}} + \Delta n\right) \cdot t^*.$$

(3) The Kepler equation for the eccentric anomaly E_k is solved by the following iterations. For $E_0 = M$, $k = 1, 2 \ldots$ do

$$E_k = M + e \sin E_{k-1}.$$

If $|E_k - E_{k-1}| \le 10^{-8}$, then stop iterations.

(4) The sine and cosine of the true anomaly f_k:

$$\sin f_k = \frac{\sqrt{1-e^2}\sin E_k}{1 - e\cos E_k}, \quad \cos f_k = \frac{\cos E_k - e}{1 - e\cos E_k}, \quad f_k = \arctan\left(\frac{\sqrt{1-e^2}\sin E_k}{\cos E_k - e}\right).$$

(5) The argument of the latitude $\varphi_k = f_k + \omega$.

(6) The second harmonic perturbations δu_k, δr_k, and δi_k of the argument of the latitude φ_k, the orbit radius r, and the angle of the inclination i_0:

$$\delta u_k = C_{\text{us}} \sin 2\varphi_k + C_{\text{uc}} \cos 2\varphi_k, \quad \delta r_k = C_{\text{rs}} \sin 2\varphi_k + C_{\text{rc}} \cos 2\varphi_k, \quad \delta i_k = C_{\text{is}} \sin 2\varphi_k + C_{\text{ic}} \cos 2\varphi_k.$$

(7) The corrected values u_k, r_k, and i_k of the argument of the latitude φ_k, the orbit radius r_k, and the angle of inclination i_k:

$$u_k = \varphi_k + \delta u_k, \quad r_k = a(1 - e\cos E_k) + \delta r_k, \quad i_k = i_0 + \delta i_k + \dot{i}_0 t^*.$$

(8) Cartesian satellite's coordinates r_1^{o} and r_2^{o} in the orbital reference frame $O\zeta$:

$$r_1^{\text{o}} = r_k \cos u_k, \quad r_2^{\text{o}} = r_k \sin u_k.$$

(9) The corrected longitude of the ascending node

$$\Omega_k = \Omega_0 + (\dot{\Omega}_0 - u)t^* - u t_{\text{oe}},$$

where $u = 7.2921151467 \cdot 10^{-5}$ (rad/s) is the WGS84 value of the Earth's rotation rate.

(10) Cartesian satellite's coordinates $R_\eta^{\text{sat}} = (R_{\eta_1}^{\text{sat}}, R_{\eta_2}^{\text{sat}}, R_{\eta_3}^{\text{sat}})^T$ in ECEF frame $O\eta$:

$$R_{\eta_1}^{\text{sat}} = r_1^{\text{o}} \cos\Omega_k - r_2^{\text{o}} \cos i_k \sin\Omega_k, \quad R_{\eta_2}^{\text{sat}} = r_1^{\text{o}} \sin\Omega_k + r_2^{\text{o}} \cos i_k \cos\Omega_k,$$
$$R_{\eta_3}^{\text{sat}} = r_2^{\text{o}} \sin i_k.$$

(11) ECEF ($O\eta$) satellite's velocities $V_\eta^{\text{sat}} = (V_{\eta_1}^{\text{sat}}, V_{\eta_2}^{\text{sat}}, V_{\eta_3}^{\text{sat}})^T$:

(a) several supplementary values are computed: derivative $\dot{f}_k$ of the true anomaly f_k, derivatives $\delta\dot{u}_k$, $\delta\dot{r}_k$, and $\delta\dot{i}_k$, of the argument of the latitude δu_k, the orbit radius r_k and the angle of the inclination i_0

$$\dot{f}_k = \frac{(\sqrt{\mu}/a^{3/2} + \Delta n)\sqrt{1-e^2}}{(1 - e\cos E_k)^2}, \quad \delta\dot{u}_k = 2(C_{\text{us}} \cos 2\varphi_k - C_{\text{uc}} \sin 2\varphi_k)\dot{f}_k,$$
$$\delta\dot{r}_k = 2(C_{\text{rs}} \cos 2\varphi_k - C_{\text{rc}} \sin 2\varphi_k)\dot{f}_k, \quad \delta\dot{i}_k = 2(C_{\text{is}} \cos 2\varphi_k - C_{\text{ic}} \sin 2\varphi_k)\dot{f}_k.$$

The moduli v_r and v_u of the satellite's range and transverse rate:

$$v_r = a\left(\sqrt{\frac{\mu}{a^{3/2}}} + \Delta n\right) \frac{e \sin E_k}{1 - e\cos E_k} + \delta\dot{r}_k, \quad v_u = r_k(\dot{f}_k + \delta\dot{u}_k);$$

(b) the components v_1^{o}, v_2^{o}, and v_3^{o} of the vector of the absolute velocity v in the orbital frame $O\zeta$:

$$v_1^{\text{o}} = v_r \cos u_k - v_u \sin u_k, \quad v_2^{\text{o}} = v_r \sin u_k + v_u \cos u_k, \quad v_3^{\text{o}} = r_k \sin f_k(\dot{i}_0 + \delta\dot{i}_k);$$

(c) the components $v_{\eta_1}^{\text{sat}}$, $v_{\eta_2}^{\text{sat}}$, and $v_{\eta_3}^{\text{sat}}$ of the vector of the absolute velocity v in ECEF frame $O\eta$:

$$\begin{aligned}
v_{\eta_1}^{\text{sat}} &= v_1^{\text{o}} \cos\Omega_k - v_2^{\text{o}} \cos i_k \sin\Omega_k + v_3^{\text{o}} \sin i_k \sin\Omega_k,\\
v_{\eta_2}^{\text{sat}} &= v_1^{\text{o}} \sin\Omega_k + v_2^{\text{o}} \cos i_k \cos\Omega_k - v_3^{\text{o}} \sin i_k \cos\Omega_k,\\
v_{\eta_3}^{\text{sat}} &= v_2^{\text{o}} \sin i_k + v_3^{\text{o}} \cos i_k;
\end{aligned}$$

(d) the components $V_{\eta_1}^{\text{sat}}$, $V_{\eta_2}^{\text{sat}}$, $V_{\eta_3}^{\text{sat}}$ of the vector of the relative velocity V in ECEF frame $O\eta$:

$$V_{\eta_1}^{\text{sat}} = v_{\eta_1}^{\text{sat}} + (u - \dot{\Omega}_0) R_{\eta_2}^{\text{sat}}, \quad V_{\eta_2}^{\text{sat}} = v_{\eta_2}^{\text{sat}} - (u - \dot{\Omega}_0) R_{\eta_1}^{\text{sat}}, \quad V_{\eta_3}^{\text{sat}} = v_{\eta_3}^{\text{sat}}.$$

Models Used to Determine Vehicle Velocity

Model of Raw Doppler Observations. A generalized model of the raw Doppler observations Z_{V_ρ} (m/s) has the form [8, 9]

$$Z_{V_\rho} = V_\rho - \lambda(f_{\Delta\tau} - f_{\Delta T}) + \delta V_{\text{ion}} + \delta V_{\text{trop}} + \delta V_{\text{mp}} + \delta V_{\text{sat}} + \delta V_{\text{rcv}} + \delta V^s, \tag{2}$$

We use here the following notation:

- λ is the radio signal wavelength;
- V_ρ is the radial velocity along the line "object–satellite" (the measured signal) specified by the expression
$$V_\rho = \frac{(R_\eta^{\text{sat}} - R_\eta)^T (V_\eta^{\text{sat}} - V_\eta)}{\rho}, \quad \rho = \sqrt{(R_\eta^{\text{sat}} - R_\eta)^T (R_\eta^{\text{sat}} - R_\eta)}; \tag{3}$$
- $f_{\Delta\tau}$ is the unknown clock drift of the receiver (estimated during processing);
- $f_{\Delta T}$ is the known clock drift of the satellite (can be compensated algorithmically);
- δV_{ion} and δV_{trop} are the errors caused by the ionospheric and tropospheric refractions (partially compensated by standard models for the ionospheric and tropospheric delays of radio signals);
- δV_{mp} is the error caused by the multipath from the obstacles surrounding the satellite antenna;
- δV_{sat} and δV_{rcv} are the instrument errors of the satellite and the receiver (sufficiently stable in time);
- δV^s is the random component in the error of Doppler observations.

In the differential mode of GPS the so-called double differences $\nabla\Delta Z_{V_{\rho_i}}$ are used for processing

$$\nabla\Delta Z_{V_{\rho_i}} = (Z_{V_{\rho_i}}^{\text{base}} - Z_{V_{\rho_i}}^{\text{rcv}}) - (Z_{V_{\rho_z}}^{\text{base}} - Z_{V_{\rho_z}}^{\text{rcv}}). \tag{4}$$

Here "base" and "rcv" indicate the observations from the base station and the rover receiver, respectively; i is the index indicating the observations from the ith satellite; z is the index indicating the observations from the zenithal satellite.

Taking into account (2), the observation (4) can be represented in the form

$$\begin{aligned}
\nabla\Delta Z_{V_{\rho_i}} &= \nabla\Delta V_{\rho_i} + \nabla\Delta V_{\text{ion}_i} + \nabla\Delta V_{\text{trop}_i} + \nabla\Delta V_{\text{mp}_i} + \nabla\Delta V_i^s,\\
\nabla\Delta V_{\rho_i} &= (V_{\rho_i^{\text{base}}} - V_{\rho_i^{\text{rcv}}}) - (V_{\rho_z^{\text{base}}} - V_{\rho_z^{\text{rcv}}}),\\
\nabla\Delta V_{(***)_i} &= \big(\delta V_{(***)_i^{\text{base}}} - \delta V_{(***)_i^{\text{rcv}}}\big) - \big(\delta V_{(***)_z^{\text{base}}} - \delta V_{(***)_z^{\text{rcv}}}\big).
\end{aligned} \tag{5}$$

Here $\nabla\Delta V_{\rho_i}$ is the differential combination of the radial velocities—the useful signal in (5).

The observation (5) has the advantage that the instrument errors of the receiver and the satellite as well as their clock errors are absent in it. In addition, the level of the residual errors $\nabla\Delta V_{\text{ion}_i}$ and $\nabla\Delta V_{\text{trop}_i}$ is lower in comparison with similar errors in the standard mode of GPS (the smaller the distance between the base station and the rover receiver and the smaller the difference between their altitudes, the lower is this level).

Determining Velocity by Processing Differential Doppler Observations. Let us compute the radial velocity $V_{\rho_i^{\rm base}}$ "satellite–base station" using ECEF coordinates and velocities of the GPS satellite and the base station ($V_\eta^{\rm base} = 0$):

$$V_{\rho_i^{\rm base}} = \frac{(R_\eta^{{\rm sat}_i} - R_\eta^{\rm base})^T}{\rho_i^{\rm base}} V_\eta^{{\rm sat}_i}. \tag{6}$$

A similar formula is used for the radial velocity $V_{\rho_i^{\rm rcv}}$ "satellite–receiver" in which the rover's velocity $V_\eta^{\rm rcv}$ is taken into account:

$$V_{\rho_i^{\rm rcv}} = V^{(1)}_{\rho_i^{\rm rcv}} + V^{(2)}_{\rho_i^{\rm rcv}}, \quad V^{(1)}_{\rho_i^{\rm rcv}} = \frac{(R_\eta^{{\rm sat}_i} - R_\eta^{\rm rcv})^T}{\rho_i^{\rm rcv}} V_\eta^{{\rm sat}_i}, \quad V^{(2)}_{\rho_i^{\rm rcv}} = -\frac{(R_\eta^{{\rm sat}_i} - R_\eta^{\rm rcv})^T}{\rho_i^{\rm rcv}} V_\eta^{\rm rcv}, \tag{7}$$

Here the component $V^{(1)}_{\rho_i^{\rm rcv}}$ can be computed directly with known coordinates and velocity of the GPS satellite and coordinates of the vehicle.

The component $V^{(2)}_{\rho_i^{\rm rcv}}$ contains information on the vehicle's velocity. Let

$$\nabla\Delta z_{V_{\rho_i}} = \nabla\Delta Z_{V_{\rho_i}} - \left[\left(V_{\rho_i^{\rm base}} - V^{(1)}_{\rho_i^{\rm rcv}}\right) - \left(V_{\rho_z^{\rm base}} - V^{(1)}_{\rho_z^{\rm rcv}}\right)\right]. \tag{8}$$

Then

$$\begin{gathered}\nabla\Delta z_{V_{\rho_i}} = -\left(V^{(2)}_{\rho_i^{\rm rcv}} - V^{(2)}_{\rho_z^{\rm rcv}}\right) + \nabla\Delta V_{{\rm ion}_i} + \nabla\Delta V_{{\rm trop}_i} + \nabla\Delta V_{{\rm mp}_i} + \nabla\Delta V_i^s = h_{(i)}^T V_\eta^{\rm rcv} + \nabla\Delta r_{\dot\rho_i}, \\ h_{(i)}^T = \left(\frac{R_\eta^{{\rm sat}_i} - R_\eta^{\rm rcv}}{\rho_i^{\rm rcv}} - \frac{R_\eta^{{\rm sat}_z} - R_\eta^{\rm rcv}}{\rho_z^{\rm rcv}}\right)^T.\end{gathered} \tag{9}$$

Here

$$\nabla\Delta r_{\dot\rho_i} = \nabla\Delta V_{{\rm ion}_i} + \nabla\Delta V_{{\rm trop}_i} + \nabla\Delta V_{{\rm mp}_i} + \nabla\Delta V_i^s$$

is the residual error of Doppler double differences.

We obtain the following linear model used to estimate V_η^M:

$$\nabla\Delta z_{V_\rho} = \begin{pmatrix} \nabla\Delta z_{V_{\rho_1}} \\ \nabla\Delta z_{V_{\rho_2}} \\ \dots \\ \nabla\Delta z_{V_{\rho_{N-1}}} \end{pmatrix} = \begin{pmatrix} h_{(1)}^T \\ h_{(2)}^T \\ \dots \\ h_{(N-1)}^T \end{pmatrix} V_\eta^{\rm rcv} + \begin{pmatrix} \nabla\Delta r_{\dot\rho_1} \\ \nabla\Delta r_{\dot\rho_2} \\ \dots \\ \nabla\Delta r_{\dot\rho_{N-1}} \end{pmatrix} = H_{(\eta)} V_\eta^{\rm rcv} + \nabla\Delta r_{\dot\rho}. \tag{10}$$

The solution of (10) by the least-squares method (under the corresponding hypotheses on noises $\nabla\Delta r_{\dot\rho}$) is of the form

$$\tilde V_\eta^{\rm rcv} = (H_{(\eta)}^T W^{-1} H_{(\eta)})^{-1} H_{(\eta)}^T W^{-1} \nabla\Delta z_{V_\rho}. \tag{11}$$

Here W is the covariance matrix of the noises $\{\nabla\Delta r_{\dot\rho_i}\}$.

Model of the Carrier Phase Observations. The model of the carrier phases observations Z_{V_φ} is as follows [8–10]:

$$Z_\varphi = \frac{\rho}{\lambda} + f(\Delta\tau - \Delta T) + N + \delta\varphi_{\rm ion} + \delta\varphi_{\rm trop} + \delta\varphi_{\rm sat} + \delta\varphi_{\rm rcv} + \delta\varphi_{\rm mp} + \delta\varphi^s. \tag{12}$$

Here ρ is the distance from the vehicle to the satellite; f is the frequency of the satellite radio signal; N is an unknown integer number (the ambiguity of the carrier phase observations); $\delta\varphi_{\rm ion}$ and $\delta\varphi_{\rm trop}$ are the errors in the carrier phase observations caused by the ionospheric and tropospheric refractions; $\delta\varphi_{\rm sat}$ and $\delta\varphi_{\rm rcv}$ are the instrument errors of the satellite and the receiver; $\delta\varphi_{\rm mp}$ is the error due to multipath; $\delta\varphi^s$ is the random component in the error of the carrier phase observations.

The single differences ∇Z_{φ_i} and ΔZ_{φ_i} of the carrier phase observations are of the form

$$\nabla Z_{\varphi_i} = Z_{\varphi_i} - Z_{\varphi_z}, \quad \Delta Z_{\varphi_i} = Z_{\varphi_i}^{\rm base} - Z_{\varphi_i}^{\rm rcv}, \tag{13}$$

where $Z_{\varphi_i}^{\mathrm{base}}$ is the carrier phase observation of the base station, $Z_{\varphi_i}^{\mathrm{rcv}}$ is the carrier phase observation of the rover receiver, and i is the satellite number.

The double difference $\nabla\Delta Z_{\varphi_i}$ takes form

$$\nabla\Delta Z_{\varphi_i} = (Z_{\varphi_i}^{\mathrm{base}} - Z_{\varphi_i}^{\mathrm{rcv}}) - (Z_{\varphi_z}^{\mathrm{base}} - Z_{\varphi_z}^{\mathrm{rcv}}). \tag{14}$$

Taking into account (12), the observation (14) can be represented as

$$\begin{aligned}
\nabla\Delta Z_{\varphi_i} &= \frac{\nabla\Delta\rho_i}{\lambda} + \nabla\Delta N_i + \nabla\Delta\varphi_{\mathrm{ion}_i} + \nabla\Delta\varphi_{\mathrm{trop}_i} + \nabla\Delta\varphi_{\mathrm{mp}_i} + \nabla\Delta\varphi_i^s, \\
\nabla\Delta\rho_i &= (\rho_i^{\mathrm{base}} - \rho_i^{\mathrm{rcv}}) - (\rho_z^{\mathrm{base}} - \rho_z^{\mathrm{rcv}}), \\
\nabla\Delta N_i &= (N_i^{\mathrm{base}} - N_i^{\mathrm{rcv}}) - (N_z^{\mathrm{base}} - N_z^{\mathrm{rcv}}), \\
\nabla\Delta\varphi_{(***)_i} &= \big(\delta\varphi_{(***)_i}^{\mathrm{base}} - \delta\varphi_{(***)_i}^{\mathrm{rcv}}\big) - \big(\delta\varphi_{(***)_z}^{\mathrm{base}} - \delta\varphi_{(***)_z}^{\mathrm{rcv}}\big).
\end{aligned} \tag{15}$$

The useful signal in the differential carrier phase observation (15) is $\nabla\Delta\rho_i/\lambda$.

The observation form (15) has the advantage that the instrument errors of the receiver and the satellites as well as their clock errors are absent in it. In addition, the level of the residual errors $\nabla\Delta\varphi_{\mathrm{ion}_i}$ and $\nabla\Delta\varphi_{\mathrm{trop}_i}$ is low (the smaller the distance between the base station and the rover receiver and the smaller the difference between their altitudes, the lower is this level).

The value of $\nabla\Delta N_i$ is an integer ambiguity of the double differences for the carrier phase observations; this ambiguity cannot be compensated with the above technique of forming the carrier phase observations.

Determining Velocity by Differential Carrier Phase Observations. The main idea of the algorithm consists in direct numerical differentiation of the carrier phases. Otherwise the algorithm differs from that described above only in the different way of formation of the measurements containing the useful information on the vehicle's motion. Therefore, we shall give just brief comments.

Let us consider the numerical derivative

$$\nabla\Delta Z_{V_{\rho_i}}^*(t_j) = \lambda \frac{\nabla\Delta Z_{\varphi_i}(t_{j+1}) - \nabla\Delta Z_{\varphi_i}(t_{j-1})}{t_{j+1} - t_{j-1}} \tag{16}$$

of the differential carrier phases $\{\nabla\Delta Z_{\varphi_i}(t_j)\}$. Using (16) we can form the approximation (estimate) of double differences of the radial velocity "receivers-satellites"

$$\nabla\Delta V_{\rho_i} = (V_{\rho_i^{\mathrm{base}}} - V_{\rho_i^{\mathrm{rcv}}}) - (V_{\rho_z^{\mathrm{base}}} - V_{\rho_z^{\mathrm{rcv}}})$$

at epoch t_j:

$$\nabla\Delta Z_{V_{\rho_i}}^*(t_j) \simeq \frac{\nabla\Delta\rho_i(t_{j+1}) - \nabla\Delta\rho_i(t_{j-1})}{t_{j+1} - t_{j-1}} \simeq \nabla\Delta V_{\rho_i}. \tag{17}$$

Then $\nabla\Delta Z_{V_{\rho_i}}^*(t_j)$ is used in the algorithm (8)–(11) instead of the similar Doppler differential observation $\nabla\Delta Z_{V_{\rho_i}}(t_j)$.

The midpoint of the algorithm is the numerical differentiation of the double differences of the carrier phases. Correct realization of the given procedure assumes absence of cycle slips in the carrier phases (change of the ambiguity values $\{\nabla\Delta N_i\}$) on the interval of differentiation. Therefore, the algorithms of detecting and excluding possible cycle slips in the carrier phases are a necessary element of the software. For example, in (10) it is possible to use the least-modulus algorithm instead of the least-squares algorithm. It should be noted that the Doppler derived velocity can also be used as additional useful information.

Mathematical Models for Determining the Acceleration

The absolute acceleration W of a vehicle is the sum of the gravity acceleration and the acceleration caused by external forces acting on the vehicle. The final aim of the problem considered below is to determine the latter acceleration.

Let us consider the Greenwich coordinate system $O\eta$, where O is the Earth's center, $O\eta_1\eta_2$ is the equatorial plane, and $O\eta_3$ is the axis of the Earth's rotation. The dynamic equation for the motion of the vehicle can be written down in the axes of this coordinate system as follows:

$$\dot{V}_\eta^{\mathrm{rcv}} = 2\hat{u}_\eta V_\eta^{\mathrm{rcv}} + g_\eta + W_\eta^{\mathrm{rcv}}, \quad g_\eta = g_{0\eta} - \hat{u}_\eta^2 R_\eta^{\mathrm{rcv}}. \tag{18}$$

Here V_η^{rcv} is the relative velocity of the vehicle in the axes of $O\eta$; $u_\eta = (0,0,u)^T$ is the vector of angular velocity of the Earth rotation; $\hat{u}_\eta$ is the skew-symmetric matrix corresponding to the vector u_η; $g_{0\eta}$ is the specific component of the gravitational force; g_η is the specific component of gravity; W_η^{rcv} is the sought-for acceleration of the vehicle in the Greenwich coordinate system.

We have

$$W_\eta^{\mathrm{rcv}} = \dot{V}_\eta^{\mathrm{rcv}} - 2\hat{u}_\eta V_\eta^{\mathrm{rcv}} - (g_{0\eta} - \hat{u}_\eta^2 R_\eta^{\mathrm{rcv}}).$$

In gravimetric applications, the Helmert formula 1 is used to determine the absolute value of normal gravity γ corrected for the flight altitude h.

In the axes of $O\eta$, we can write down

$$g_\eta = B\begin{pmatrix} 0 \\ 0 \\ -\gamma \end{pmatrix}, \quad B = \begin{pmatrix} -\sin\lambda & -\cos\lambda\sin\varphi & \cos\lambda\cos\varphi \\ \cos\lambda & -\sin\lambda\sin\varphi & \sin\lambda\cos\varphi \\ 0 & \cos\varphi & \sin\varphi \end{pmatrix},$$

where B is the matrix of the relative orientation for the Greenwich and geographical coordinate system and λ and φ are the geographical coordinates of the object.

Let us assume that the radial acceleration A_{ρ_i} along the line "object–navigation satellite" is known:

$$\begin{aligned} A_{\rho_i} &= \frac{(V_\eta^{\mathrm{sat}_i} - V_\eta^{\mathrm{rcv}})^T (V_\eta^{\mathrm{sat}_i} - V_\eta^{\mathrm{rcv}})}{\rho_i^{\mathrm{rcv}}} - \frac{\dot{\rho}_i^{\mathrm{rcv}2}}{\rho_i^{\mathrm{rcv}}} + \frac{(R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}})^T}{\rho_i^{\mathrm{rcv}}} (\dot{V}_\eta^{\mathrm{sat}_i} - \dot{V}_\eta^{\mathrm{rcv}}), \\ \rho_i^{\mathrm{rcv}} &= \sqrt{(R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}})^T (R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}})}. \end{aligned} \tag{19}$$

Here $R_\eta^{\mathrm{sat}_i}$ and $V_\eta^{\mathrm{sat}_i}$ are the Greenwich coordinates and the vector of the relative velocity of the ith navigation satellite; R_η^{rcv} and V_η^{rcv} are the Greenwich coordinates and the vector of the relative velocity of the object; ρ_i^{rcv} is the distance between the satellite and the object.

The radial acceleration A_{ρ_i} can be represented as the sum of two components $A_{\rho_i}^{(\mathrm{I})}$ and $A_{\rho_i}^{(\mathrm{II})}$, where

$$\begin{aligned} A_{\rho_i}^{(\mathrm{I})} &= \frac{(V_\eta^{\mathrm{sat}_i} - V_\eta^{\mathrm{rcv}})^T (V_\eta^{\mathrm{sat}_i} - V_\eta^{\mathrm{rcv}})}{\rho_i} - \frac{\dot{\rho}_i^{\mathrm{rcv}2}}{\rho_i} + \frac{(R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}})^T}{\rho_i^{\mathrm{rcv}}} \dot{V}_\eta^{\mathrm{sat}_i}, \\ A_{\rho_i}^{(\mathrm{II})} &= -\frac{(R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}})^T}{\rho_i^{\mathrm{rcv}}} \dot{V}_\eta^{\mathrm{rcv}}. \end{aligned} \tag{20}$$

The first component $A_{\rho_i}^{(\mathrm{I})}$ can be calculated explicitly on the basis of known data on coordinates and velocities for the satellite and the object. The component $A_{\rho_i}^{(\mathrm{II})}$ contains information on $\dot{V}_\eta^{\mathrm{rcv}}$ and on the sought-for acceleration W_η^{rcv}.

Let us assume that we can estimate A_{ρ_i} on the basis of raw GPS observations:

$$Z_{A_{\rho_i}} = A_{\rho_i} + r_{\ddot{\rho}_i}.$$

Here $r_{\ddot{\rho}_i}$ is the generalized error in the found value of A_{ρ_i}.

Let $z_{A_{\rho_i}} = Z_{A_{\rho_i}} - A_{\rho_i}^{(\mathrm{I})}$. Then $\dot{V}_\eta$ can be determined using the following model for the estimation problem:

$$
\begin{aligned}
z_{A_\rho} &= \begin{pmatrix} z_{A_{\rho_1}} \\ z_{A_{\rho_2}} \\ \dots \\ z_{A_{\rho_{(N)}}} \end{pmatrix} = \begin{pmatrix} h_{(1)}^T \\ h_{(2)}^T \\ \dots \\ h_N^T \end{pmatrix} \dot{V}_\eta^{\mathrm{rcv}} + \begin{pmatrix} r_{\ddot{\rho}_1} \\ r_{\ddot{\rho}_2} \\ \dots \\ r_{\ddot{\rho}_N} \end{pmatrix} = H\dot{V}_\eta^{\mathrm{rcv}} + r_{\ddot{\rho}}, \\
h_{(i)}^T &= -\frac{R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}}}{\rho_i}.
\end{aligned} \tag{21}
$$

Here $r_{\ddot{\rho}}$ is the generalized error of observations and N is the number of visible satellites.

The solution to (21) obtained by the least-squares method (under the corresponding hypotheses on noises $r_{\ddot{\rho}_i}$ in observations) is of the form

$$
\hat{\dot{V}}_\eta^{\mathrm{rcv}} = (H^T Q^{-1} H)^{-1} H^T Q^{-1} z_{A_\rho},
$$

where Q is the covariance matrix of noises $r_{\ddot{\rho}_i}$.

Using (18), we get

$$
\tilde{W}_\eta^{\mathrm{rcv}} = \hat{\dot{V}}_\eta^{\mathrm{rcv}} - 2\hat{u}_\eta V_\eta^{\mathrm{rcv}} - (g_{0\eta} - \hat{u}_\eta^2 R_\eta^{\mathrm{rcv}}). \tag{22}
$$

Determining Acceleration by Processing the Doppler Observations. Let us consider three values of double differences $\nabla\Delta Z_{V_{\rho_i}}(t_{j-1})$, $\nabla\Delta Z_{V_{\rho_i}}(t_j)$, and $\nabla\Delta Z_{V_{\rho_i}}(t_{j+1})$ for Doppler observations. Using the first-order central difference of these values, we can form an approximation (estimate) $\nabla\Delta Z_{A_{\rho_i}}(t)$ for the first derivative of $\nabla\Delta Z_{V_{\rho_i}}(t)$ at the time instant t_j:

$$
\nabla\Delta Z_{A_{\rho_i}}(t_j) = \frac{\nabla\Delta Z_{V_{\rho_i}}(t_{j+1}) - \nabla\Delta Z_{V_{\rho_i}}(t_{j-1})}{t_{j+1} - t_{j-1}}. \tag{23}
$$

The estimate $\nabla\Delta Z_{A_{\rho_i}}(t_j)$ is a useful signal for the following double difference

$$
\nabla\Delta A_{\rho_i}(t_j) = (A_{\rho_i^{\mathrm{base}}}(t_j) - A_{\rho_i^{\mathrm{rcv}}}(t_j)) - (A_{\rho_z^{\mathrm{base}}}(t_j) - A_{\rho_z^{\mathrm{rcv}}}(t_j)).
$$

Let us introduce the model

$$
\nabla\Delta Z_{A_{\rho_i}} = \nabla\Delta A_{\rho_i} + \nabla\Delta r_{\ddot{\rho}_i},
$$

where $\nabla\Delta r_{\ddot{\rho}_i}$ is the generalized error in the double difference of radial accelerations.

Taking into account (19) and (20), the double difference $\nabla\Delta A_{\rho_i}$ can be represented as the sum of two components:

$$
\begin{gathered}
\nabla\Delta A_{\rho_i} = \nabla\Delta A_{\rho_i}^{(\mathrm{I})} + \nabla\Delta A_{\rho_i}^{(\mathrm{II})}, \\
\nabla\Delta A_{\rho_i}^{(\mathrm{I})} = \big(A_{\rho_i^{\mathrm{base}}} - A_{\rho_i^{\mathrm{rcv}}}^{(\mathrm{I})}\big) - \big(A_{\rho_z^{\mathrm{base}}} - A_{\rho_z^{\mathrm{rcv}}}^{(\mathrm{I})}\big), \quad \nabla\Delta A_{\rho_i}^{(\mathrm{II})} = -\big(A_{\rho_i^{\mathrm{rcv}}}^{(\mathrm{II})} - A_{\rho_z^{\mathrm{rcv}}}^{(\mathrm{II})}\big).
\end{gathered}
$$

The first component is explicitly calculated with the use of the known data on coordinates and velocities of the navigation satellites, the rover receiver, and the base station. The second component contains the unknown parameter $\dot{V}_\eta^{\mathrm{rcv}}$ to be estimated. Let

$$
\nabla\Delta z_{A_{\rho_i}} = \nabla\Delta Z_{A_{\rho_i}} - \nabla\Delta A_{\rho_i}^{(\mathrm{I})}.
$$

Then the following model is valid for $\nabla\Delta z_{A_{\rho_i}}$:

$$
\nabla\Delta z_{A_{\rho_i}} = h_{(i)}^T \dot{V}_\eta^{\mathrm{rcv}} + \nabla\Delta r_{\ddot{\rho}_i}, \quad h_{(i)}^T = \left(\frac{R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}}}{\rho_i^{\mathrm{rcv}}} - \frac{R_\eta^{\mathrm{sat}_z} - R_\eta^{\mathrm{rcv}}}{\rho_z^{\mathrm{rcv}}}\right)^T. \tag{24}
$$

In the case of $N-1$ observations (N is the number of visible satellites), we obtain the following linear model for the problem of estimating the value of $\dot{V}_\eta^{\mathrm{rcv}}$:

$$z_{A_\rho} = \begin{pmatrix} z_{A_{\rho_1}} \\ z_{A_{\rho_2}} \\ \dots \\ z_{A_{\rho_{N-1}}} \end{pmatrix} = \begin{pmatrix} h_{(1)}^T \\ h_{(2)}^T \\ \dots \\ h_N^T \end{pmatrix} \dot{V}_\eta^{\mathrm{rcv}} + \begin{pmatrix} \nabla\Delta r_{\ddot{\rho}_1} \\ \nabla\Delta r_{\ddot{\rho}_2} \\ \dots \\ \nabla\Delta r_{\ddot{\rho}_{N-1}} \end{pmatrix} = H\dot{V}_\eta^{\mathrm{rcv}} + \nabla\Delta r_{\ddot{\rho}}. \tag{25}$$

The solution to (25) obtained by the least-squares method (under the corresponding hypotheses on noises $\nabla\Delta r_{\ddot{\rho}}$) is of the form

$$\hat{\dot{V}}_\eta^{\mathrm{rcv}} = (H^T Q^{-1} H)^{-1} H^T Q^{-1} \nabla\Delta z_{A_\rho}.$$

Here Q is the covariance matrix of noises $\nabla\Delta r_{\ddot{\rho}_i}$.

Finally, we use Eq. (22) to estimate the acceleration $\tilde{W}_\eta^{\mathrm{rcv}}$.

Determining Acceleration with Differential Carrier Phase Observations. The algorithm considered below differs from that presented above only in another way of forming the observations with useful information on the vehicle acceleration. Therefore, we restrict ourselves to brief comments.

Let us consider the three values $\nabla\Delta Z_{\varphi_i}(t_{j-1})$, $\nabla\Delta Z_{\varphi_i}(t_j)$, and $\nabla\Delta Z_{\varphi_i}(t_{j+1})$ of differential carrier phase observations. Numerical differentiation is performed using the second-order central differences. As a result, we obtain the approximation (estimate) $\nabla\Delta Z_{A_{\rho_i}}$ (see (15)) for the double differences

$$\nabla\Delta A_{\rho_i} = (A_{\rho_i^{\mathrm{base}}} - A_{\rho_i^{\mathrm{rcv}}}) - (A_{\rho_z^{\mathrm{base}}} - A_{\rho_z^{\mathrm{rcv}}})$$

of radial accelerations along the lines "receivers–satellites" at the time instant t_j:

$$\nabla\Delta Z_{A_{\rho_i}}(t_j) = \lambda \frac{\nabla\Delta Z_{\varphi_i}(t_{j+1}) - 2\nabla\Delta Z_{\varphi_i}(t_j) + \nabla\Delta Z_{\varphi_i}(t_{j-1})}{\Delta t^2}. \tag{26}$$

Here $\Delta t = t_{j+1} - t_j = t_j - t_{j-1}$.

As in the case of Doppler observations, we introduce

$$\nabla\Delta z_{A_{\rho_i}} = \nabla\Delta Z_{A_{\rho_i}} - \nabla\Delta A_{\rho_i}^{(\mathrm{I})},$$

where

$$\nabla\Delta A_{\rho_i}^{(\mathrm{I})} = \big(A_{\rho_i^{\mathrm{base}}} - A^{(\mathrm{I})}_{\rho_i^{\mathrm{rcv}}}\big) - \big(A_{\rho_z^{\mathrm{base}}} - A^{(\mathrm{I})}_{\rho_z^{\mathrm{rcv}}}\big).$$

The components $A_{\rho_{(*)}^{\mathrm{base}}}$ and $A^{(\mathrm{I})}_{\rho_{(*)}^{\mathrm{rcv}}}$ are specified in (19) and (20).

According to (24) and (25), the model for the problem of estimating the vector $\dot{V}_\eta$ takes the form

$$\nabla\Delta z_{A_\rho} = \begin{pmatrix} \nabla\Delta z_{A_{\rho_1}} \\ \nabla\Delta z_{A_{\rho_2}} \\ \dots \\ \nabla\Delta z_{A_{\rho_{N-1}}} \end{pmatrix} = \begin{pmatrix} h_{(1)}^T \\ h_{(2)}^T \\ \dots \\ h_{(N-1)}^T \end{pmatrix} \dot{V}_\eta^{\mathrm{rcv}} + \begin{pmatrix} \nabla\Delta r_{\ddot{\varphi}_1} \\ \nabla\Delta r_{\ddot{\varphi}_2} \\ \dots \\ \nabla\Delta r_{\ddot{\varphi}_{N-1}} \end{pmatrix} = H\dot{V}_\eta^{\mathrm{rcv}} + \nabla\Delta r_{\ddot{\varphi}},$$
$$h_{(i)}^T = \left(\frac{R_\eta^{\mathrm{sat}_i} - R_\eta^{\mathrm{rcv}\prime}}{\rho_i^{\mathrm{rcv}\prime}} - \frac{R_\eta^{\mathrm{sat}_z} - R_\eta^{\mathrm{rcv}\prime}}{\rho_z^{\mathrm{rcv}\prime}} \right)^T. \tag{27}$$

Here $\nabla\Delta r_{\ddot{\varphi}_i}$ is the total error of observations due to numerical differentiation of errors in carrier phase observations.

In its structure, the problem (27) completely coincides with the model (25) for the problem of determining the acceleration by Doppler observations. In order to solve (27), we can also use the least-squares method under the corresponding hypotheses on the characteristics of errors $\nabla\Delta r_{\ddot{\varphi}}$.

The algorithm for determining acceleration uses direct numerical differentiation of the double differences of carrier phases. Similar remarks can be formulated concerning algorithms for cycle slip detection and exclusion as in the velocity determination problem.

It should be noted that Doppler-derived acceleration can be used as additional useful information when processing phase observations.

Determining Location with Differential Carrier Phase Observations

Let us return to the main problem of satellite navigation—the problem of determining the coordinates of a vehicle by differential carrier phase observations. Let us assume that the GPS derived velocity is already obtained in postprocessing.

The proposed approach to the problem of determining the vehicle's location is rather obvious. We introduce a kinematic model of the vehicle motion:

$$\dot{R}_{\eta}^{\mathrm{rcv}'} = \tilde{V}_{\eta}^{\mathrm{rcv}}, \tag{28}$$

where $R_{\eta}^{\mathrm{rcv}'}$ are the simulated coordinates of the object, and $\tilde{V}_{\eta}^{\mathrm{rcv}}$ are the GPS-derived velocities based on processing differential Doppler or carrier phase observations.

Let us introduce the errors of the model (28):

$$\Delta R_{\eta} = R_{\eta}^{\mathrm{rcv}'} - R_{\eta}^{\mathrm{rcv}}, \quad \delta V_{\eta} = \tilde{V}_{\eta}^{\mathrm{rcv}} - V_{\eta}^{\mathrm{rcv}},$$

where R_{η}^{rcv}, V_{η}^{rcv} are true coordinates and velocities of the vehicle, ΔR_{η} is the error in location, and δV_{η} is the error in GPS derived velocity. We can write

$$\Delta\dot{R}_{\eta} = \delta V_{\eta}. \tag{29}$$

Now we can linearize the differential carrier phase observations (15) around the model (29) of the vehicle motion. The linearized equations will take form

$$\nabla\Delta z(t_j) = \begin{pmatrix} h_{(1)}^T \\ h_{(2)}^T \\ \dots \\ h_{(N-1)}^T \end{pmatrix} \Delta R_{\eta}(t_j) + \begin{pmatrix} \nabla\Delta N_1 \\ \nabla\Delta N_2 \\ \dots \\ \nabla\Delta N_{N-1} \end{pmatrix} + \begin{pmatrix} \nabla\Delta r_1 \\ \nabla\Delta r_2 \\ \dots \\ \nabla\Delta r_{N-1} \end{pmatrix} = H(t_j)\Delta R_{\eta}(t_j) + \nabla\Delta N + \nabla\Delta r(t_j), \tag{30}$$

where the rows $h_{(i)}^T$ of the matrix $H(t_j)$ are defined in (27), $\nabla\Delta N$ is the vector of integer ambiguities, and $\nabla\Delta r$ is the vector of total errors of the carrier phase observations.

Let us introduce the state vector

$$x_j = (\Delta R_{\eta}{}^T(t_j), \nabla\Delta N^T)^T$$

and transform the continuous time model (29) to its discrete form. Then the problem of estimation of the state vector $x_j = x(t_j)$ using differential carrier phase observations $\nabla\Delta z_j = \nabla\Delta z(t_j)$ can be set as

$$x_{j+1} = x_j + q_j, \quad \nabla\Delta z_j = H_j x_j + r_j, \tag{31}$$

where $H_j = H(t_j)$, $r_j = \nabla\Delta r(t_j)$, and q_j is the system driving noise, equivalent to δV_{η} in the discrete form (29). A Kalman smoothing filter can be used to solve (31) under appropriate hypotheses on the noises q_j, r_j.

A peculiarity of the proposed algorithm lies in the necessity to parametrize the system driving noise δV_{η} model. Both heuristic models and models based on statistical analysis of the corresponding carrier phase residuals can be used here.

The advantage of the proposed approach lies in the significant simplification of detecting and excluding the carrier phase cycle slips. This is because we reduce this problem to the simpler case of obtaining the velocity.

Experience shows that the proposed algorithms are effective even with large >100 km base line lengths.

Conclusion

The problem of how to determine velocity, acceleration, and position by raw Doppler and carrier phase GPS observations was discussed. Corresponding mathematical models for differential Doppler and carrier phase GPS observations were described. An approach was proposed for vehicle velocity and acceleration determination. This approach is based on direct numerical differentiation of raw carrier phase observations. The problem was reduced to a linear estimation. Application of the proposed algorithms in airborne gravimetry [1, 3] shows their efficiency.

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