MATHEMATICAL MODEL OF ULTRARELATIVISTIC ELECTRON PASSAGE THROUGH THICK CRYSTAL INCLUDING AXIAL CHANNELING, MULTIPLE SCATTERING AND γ-QUANTA RADIATION

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High energy electron beam passage through the single crystal is considered to be homogeneous Markov process with diffusion and jump components corresponding to multiple scattering and γ-quanta radiation respectively.

From computational point of view, solution of the following system of stochastic differential equations has to be constructed. Initial and reflection condition on the boundary of elementary cell are given

$$dr_{\perp}=v_{\perp}\frac{ds}{\lambda(r,E)}; \qquad dE=\int H(u,E,r)\nu(ds,du);$$

$$dv_{\perp}=-\frac{\nabla_r U(r)ds}{m(E)\lambda(r,E)}+\frac{\sigma_p(r,E)dw(s)}{m(E)\sqrt{\lambda(r,E)}}-2I_{\Gamma}(r)(n(r),v_{\perp})v_{\perp},$$

where $r_{\perp}(t)$, $v_{\perp}(t)$, $E(t)$ are to symbolize transverse coordinate, transverse velocity and total energy of a particle respectively, $m=E/c^2$, $U(r)$ - an averaged potential of atomic string, $\lambda(r,E)$ - an integral intensity of radiation, Γ denotes the boundary of elementary cell, $n(r)$ stands for internal normal, $w(s)$ is a standard two-dimensional Wiener process independent, with Poisson point measure ν, defined on $[0,\infty]\times[0,1]$, $E\nu(ds,du)=dsdu$, where s denotes a relative time (dimensionless parameter representing expected average number of emissions), u is a γ-quantum energy.

This system has a number of mathematical properties, making it very useful for numerical analysis. Thus, electron transverse velocity doesn't change during γ-quantum emission and the diffusion coefficient $\sigma_p(r,E)$, characterizing multiple scattering, in turn, doesn't depend on transverse velocity. All the physical characteristics of interest can be calculated as functionals of Markov process trajectories, i.e. physical time $dt=ds/\lambda(r,E)$.

Numerical results for Ge single crystal with modified Molier potential are presented. It's necessary to note that intensity of emission can increase as a result of transverse energy decreasing during emission and capturing particle in channel. Thus, regeneration feedback leading to self accelerating of emission, described in a literature, can arise.