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THE CONFERENCE "DYNAMICS IN SIBERIA", NOVOSIBIRSK, FEBRUARY 26 – MARCH 4, 2017

I.A. DYNNIKOV, A.A. GLUTSYUK, A.E. MIRONOV, I.A. TAIMANOV, A.YU. VESNIN

ABSTRACT. In this article abstracts of talks of the Conference "Dynamics in Siberia" held in Sobolev Institute of Mathematics, February 26 – March 4, 2017 are presented.

Keywords: dynamical systems, geometry, integrable systems, mathematical physics.

The second international conference "Dynamics in Siberia" was held at the Sobolev Institute of Mathematics SB RAS (Novosibirsk) from February 26 to March 4, 2017 (for information on the previous conference see [1]). Members of the program committee were as follows: I.A. Dynnikov, A.A. Glutsyuk, A.E. Mironov, I.A. Taimanov and A.Yu. Vesnin.

More than 50 experts on dynamical systems, mathematical physics, geometry and topology participated in the conference. The conference program consisted of plenary talks and short talks. The talks were made by well-known experts from Moscow, St. Petersburg, Novosibirsk, Chelyabinsk, Gorno-Altaisk, Nizhny Novgorod, Grozny, Kemerovo, Krasnoyarsk, Tomsk, Ufa, Magadan, Yakutsk and also by well-known mathematicians from China, France, Germany, Italy, Japan, Taiwan, Belarus. More than 20 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

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The absolute boundary of discrete Heisenberg group

A. Malyutin (St. Petersburg)

A.M.Vershik introduced the notion of absolute boundary (also called the 'absolute') for finitely generated groups. The absolute boundary of a group is a topological space that can be regarded as the boundary at infinity (Dynkin's exit-boundary) of the so-called dynamical graph over the Cayley graph of the group. The absolute boundary contains, in a sense, the Poisson-Furstenberg boundary of the group and is contained in the Martin boundary of the dynamical graph. A part of the absolute boundary can be identified with the set of all minimal positive eigenfunctions of the Laplacian determined by the simple random walk on the group. The absolute boundary of an abelian group is homeomorphic to a closed ball of certain dimension. (The fact that the absolute boundary of the infinite cyclic group is an interval is a reformulation of de Finetti's theorem.) The absolute boundary of the free non-abelian group is homeomorphic to the direct product of the Cantor set by an interval. The next phase in developing the theory of absolute boundary is the case of nilpotent groups. We show that in the case of discrete Heisenberg group with the standard generating set, the absolute boundary is homeomorphic to the disjoint union of a closed 2-disk and a countable set of isolated points whose limit set is the boundary of the 2-disk. In order to find the absolute we need, in particular, to describe the set of all geodesic rays in the (Cayley graph of) Heisenberg group.

Naturally graded Lie algebras of slow growth

D. Millionshchikov (Moscow)[5]

The growth of a finitely generated infinite-dimensional Lie algebra $\mathfrak{g}$ can be described by the Gelfand-Kirillov dimension which is defined as

$$GK\dim\mathfrak{g} = \limsup_{n\to\infty}\frac{\log\dim V^n}{\log n},$$

where V^n is the subspace in $\mathfrak{g}$ spanned by all elements of length at most n with arbitrary arrangements of brackets. A finite Gelfand-Kirillov dimension means that there exists a polynomial $P(x)$ such that $\dim V^n{<}P(n)$ for all $n{>}1$. Shalev and Zelmanov obtained [6] important results on Lie algebras of linear growth (obviously they have the GK-dimension equal to 1). Petrograsky constructed examples of Lie algebras $\mathfrak{g}$ with non-linear growth but however such that $GK\dim\mathfrak{g} = 1$ [5]. Shalev, Zelmanov defined Lie algebras of maximal class – a special subclass of the positively graded two-generated Lie algebras with the slowest possible growth ($\dim V^n = n{+}1$).

Kac [2] classified under a certain technical condition infinite-dimensional $\mathbb{Z}$-graded simple Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{Z}}\mathfrak{g}_i$ of finite growth in a following sense: $\dim\mathfrak{g}_n \le P(n)$ for some polynomial $P(x)$. Moreover, Kac conjectured that dropping the condition would add only the Witt algebra. Finally Kac's conjecture was proved in 1990 by Mathieu [3].

Fialowski [1] classified $\mathbb{N}$-graded Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{N}}\mathfrak{g}_i$ with one-dimensional homogeneous components $\mathfrak{g}_i$ that are multiplicatively generated by two elements from $\mathfrak{g}_1$ and $\mathfrak{g}_2$ respectively. Besides other algebras one can find in her list the positive part of the Witt (Virasoro) Lie algebra W^+ and two positively graded Lie

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algebras $\mathfrak{n}_1$ and $\mathfrak{n}_2$ that are maximal nilpotent subalgebras of twisted loop algebras $A_1^{(1)}$ and $A_2^{(2)}$ respectively.

A Lie algebra $\mathfrak{g}$ is called naturally graded if it is isomorphic to its associated graded Lie algebra $\mathrm{gr}_C\mathfrak{g}$ with respect to the filtration by ideals $C^i\mathfrak{g}$ of the descending central sequence. For instance $\mathrm{gr}_C W^+ \cong \mathfrak{m}_0$, where $\mathfrak{m}_0$ can be defined by its infinite basis $e_1, e_2, \dots,$ and structure relations: $[e_1, e_i] = e_{i+1}, i = 2, 3, \dots$ $[e_k, e_l] = 0, k, l \geq 2$. It was proved by Vergne [7] that up to an isomorphism there is the only one naturally graded Lie algebra of maximal class and it is $\mathfrak{m}_0$.

We classify naturally graded Lie algebras with the linear growth $\dim V^n \leq \frac{3}{2}n{+}const$.

Theorem. Let $\mathfrak{g} = \oplus_{i=1}^{+\infty}\mathfrak{g}_i$ be a real naturally graded Lie algebra such that:

$$\dim \mathfrak{g}_i + \dim \mathfrak{g}_{i+1} \leq 3, \forall i \in \mathbb{N}.$$

Then $\mathfrak{g} = \oplus_{i=1}^{+\infty}\mathfrak{g}_i$ is isomorphic to the only one Lie algebra from the following list:

$$\mathfrak{m}_0, \mathfrak{n}_1^{\pm},\ \mathfrak{n}_2, \left\{\mathfrak{m}_0^S \mid S \subset \{3,5,7,9,\dots\}\right\},$$

where $\mathfrak{n}_1^{\pm}$ are special subalgebras in loop Lie algebras $\mathfrak{so}(3,\mathbb{R})$ and $\mathfrak{so}(1,2,\mathbb{R})$ respectively and $\left\{\mathfrak{m}_0^S \mid S \subset \{3,5,7,9,\dots\}\right\}$ are central extensions of $\mathfrak{m}_0$.

Bibliography

[1] Fialowski A. On the classification of graded Lie algebras with two generators // Moscow Univ. Math. Bull. 1983. V.38. N 2, P. 76–79.

[2] Kac V.G. Simple graded Lie algebras of finite growth// Math. USSR Izv.1968. V. 2, P. 1271–1311.

[3] Mathieu O. Classification of simple graded Lie algebras of finite growth// Invent. Math. 1990. V. 108, P. 455–519.

[4] Millionschikov D.V. Graded filiform Lie algebras and symplectic nilmanifolds // Geometry, topology, and mathematical physics. Provid ence (RI): Amer. Math. Soc., 2004. P. 259–279. (AMS Transl. Ser.2; V. 212).

[5] Petrogradsky V. Nil Lie p-algebras of slow growth// Communications in Algebra. 2017, V. 45, N. 7. P. 2912–294.

[6] Shalev A., Zelmanov E.I. Narrow algebras and groups // J. of Math. Sciences. 1999. V. 93, N 6. P. 951–963.

[7] Vergne M. Cohomologie des algèbres de Lie nilpotentes // Bull. Soc. Math. France. 1970. V. 98. P. 81–116.

On Hamiltonian geometry of associativity equations

N. Pavlenko (Moscow)[6]

In the case of three primary fields, the associativity equations or the Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations of the two-dimensional topological quantum field theory can be represented as integrable nondiagonalizable systems of hydrodynamic type (O.I. Mokhov, [1]). After that the question about the Hamiltonian nature of such hydrodynamic type systems arose. O.I. Mokhov and E.V. Ferapontov [2] have shown that the Hamiltonian geometry of these systems essentially depends on the metric of the associativity equations. Namely there are examples of the WDVV equations which are equivalent to the hydrodynamic type systems with local homogeneous first-order Dubrovin-Novikov type Hamiltonian structures,

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