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Influence of Quantum Decoherence on Collective Neutrino Oscillations in the Model of Homogeneous Neutrino Gas

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Abstract—Collective oscillations and quantum decoherence in neutrino physics are two rapidly developing lines of research that act as gateways to a potentially new physics. Moreover, they promise to shed light on still unresolved problems, particularly the neutrino mass hierarchy. In this paper we study the interplay of these two effects, namely we consider the possible suppression effect on the collective oscillations when neutrinos are viewed in the formalism of open quantum systems, i.e., when quantum decoherence is taken into account. We consider a model of a homogeneous neutrino gas to take into account angular asymmetries, which play a special role in the formation of collective unstable modes. Using numerical analyses of the emerging slow modes, we find that in the case of neutrino fluxes from supernovae there is a potential to differentiate between two hierarchies of neutrino masses by taking into account the dissipative effect of quantum decoherence on collective oscillations.

Keywords: neutrino, neutrino quantum decoherence, neutrino oscillations

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The mass ordering of neutrinos is one of the major unresolved problems in the field of neutrino physics. One of the promising phenomena that could shed some light on this issue is collective oscillations arising from the interactions between the neutrinos themselves. Despite the intense efforts in this relatively new field, the study of collective oscillations is still an extremely difficult task due to the nonlinearity of the resulting equations of motion. In this regard, simplified models have been developed, such as the "light-bulb" model, which, although providing a number of predictions on the collective behavior of neutrinos in supernova fluxes, still cannot describe a number of effects related to the angular distribution of neutrinos of different flavours. The incorporation of the latter into the consideration of neutrino evolution has led to the discovery of a number of new types of collective oscillations under the general term "fast oscillations." Their characteristic feature is the rapid growth of flavour conversion independent of the vacuum mixing parameters. For a recent review of collective oscillations see, e.g. [1].

Apart from more accurate portrayal of neutrino propagation, another key factor may be the phenomenon of quantum decoherence which is present in the models where neutrino is considered to be an open quantum system. Generally speaking, quantum decoherence is a destruction of the quantum superposition of neutrino mass states via the interaction with the reservoir. The source of such an effect might be the interaction of neutrinos with the fluctuating medium and with the fluctuating magnetic field [2, 3], as well as the interaction with the fluctuating gravitational field [4]. Another proposed mechanism is the neutrino decay into a lighter state and a massless particle, as well as the inverse process [5]. Usually, this loss of quantum coherence does not qualitatively disturb the flavor conversion process. However in the case of collective oscillations coherence is the leading factor as it is relevant to the neutrino—neutrino interaction and it was shown in our previous works that quantum decoherence can in theory fully suppress the collective behavior [6, 7].

In the present work we consider the evolution of ultrarelativistic neutrinos of three flavours with quantum decoherence at some large distance from the neutrinosphere surface ($\sim 10^2$ km), where the collective behavior might occur. We have analyzed the influence of the quantum decoherence on collective oscillations before within a one-angle approximation using the "light-bulb" model. Yet, in general one needs to worry about the situations where neutrinos

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stream in all directions. One can model that by thinking in terms of an interacting neutrino gas that is roughly homogeneous at various scales.

Consequently, in present work we consider a homogeneous neutrino gas described by a toy-model of "colliding beams." In this model we essentially have four beams: a right moving neutrino beam (index r) and a right moving antineutrino beam ($\bar{r}$) separated by the relative angle α interacting with a left moving neutrino beam (l) and a left moving antineutrino beam ($\bar{l}$). Each one is characterized by the appropriate EoM and by the neutrino density $g_i, i \in \{r, l, \bar{r}, \bar{l}\}$. The latter are assumed to be constant and are normalized by the following condition: $|g_l| + |g_r| + |g_{\bar{l}}| + |g_{\bar{r}}| = 2$. These densities can be conveniently expressed in terms of symmetry parameters: a, which accounts for the neutrino-antineutrino asymmetry, and b, which accounts for the left-right asymmetry. With these conditions one can employ the following:

$$\begin{aligned} g_l &= \frac{1}{2}(1+a)(1-b), \\ g_r &= \frac{1}{2}(1+a)(1+b), \\ g_{\bar{l}} &= -\frac{1}{2}(1-a)(1+b), \\ g_{\bar{r}} &= -\frac{1}{2}(1-a)(1-b). \end{aligned} \tag{1}$$

We describe the evolution of the neutrinos and antineutrinos in the flavor basis in terms of the density matrices $\rho(t)$ and $\bar{\rho}(t)$, respectively, using the Lindblad equation:

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + D[\rho(t)]. \tag{2}$$

A similar equation holds for the antineutrino evolution. The total Hamiltonian of the neutrino H includes the usual three contributions from neutrino masses, background matter, and from other neutrinos, $H = H_{\text{vac}} + H_\lambda + H_{\nu\nu}$.

In the present study we include the neutrino mixing as we are interested primarily in the "slow oscillations" driven by the neutrino masses. Thus, the vacuum term, H_{vac}, for a neutrino mode of energy E in the flavor basis using the neutrino mixing matrix U reads as follows

$$\begin{aligned} H_{\text{vac}} &= \frac{UM^2U^\dagger}{2E}, \\ M^2 &:= \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2). \end{aligned} \tag{3}$$

Here $\Delta m_{kl} := m_k^2 - m_l^2$ denotes square mass differences of the neutrinos. For convenience we have also introduced a mass-squared difference ratio $\eta = \Delta m_{31}^2/\Delta m_{21}^2$ between atmospheric and solar mass differences.

The second term, H_λ, describes neutrino interactions with the background matter and in the case of the core-collapsed supernova is dominated by the charged current interaction of electron neutrinos with electrons of a net density N_e. In the weak interaction basis it can be written as

$$H_\lambda = \sqrt{2}G_{\text{F}}\,\text{diag}(N_e, 0, 0), \tag{4}$$

where G_{F} is the Fermi coupling constant. Here, we also define the effective MSW-potential strength as $\lambda = \sqrt{2}G_{\text{F}}N_e$.

The neutrino–neutrino interaction is depended on the velocities of the interacting modes. As neutrinos are considered to be ultrarelativistic, the four velocity vector is denoted by $v^\mu = (1, \mathbf{v})$ and in our simple model $|\mathbf{v}| = 1$. Hence, the interaction between a neutrino and other neutrinos of N discrete modes with velocities $\mathbf{v}_j$ reads

$$H_{\nu\nu} := \sqrt{2}G_{\text{F}}n_\nu \sum_{j=1}^{N}(1 - \mathbf{v}\mathbf{v}_j)\rho. \tag{5}$$

This part of the Hamiltonian is also proportional to the effective neutrino density $n_\nu := \frac{1}{2}(n_{\nu_e} - n_{\bar{\nu}_e} + n_{\nu_x} - n_{\bar{\nu}_x} + n_{\nu_y} - n_{\bar{\nu}_y})$ and the relative angle of the propagation direction of the neutrino modes. In addition, here we have introduced the notion of the effective neutrino–neutrino potential $\mu = \sqrt{2}G_{\text{F}}n_\nu$ for later evaluation.

The last term in the Eq. (2) is a dissipator, $D[\rho]$, which describes the quantum decoherence of the neutrino mass states and is given by the expression

$$D[\rho] = \frac{1}{2}\sum_{k=1}^{8}\left[V_k, \rho V_k^\dagger\right] + \left[V_k\rho, V_k^\dagger\right], \tag{6}$$

where V_k are dissipative operators corresponding to the interaction of the neutrino with the reservoir.

Following the approach that is widely used in studies of collective neutrino oscillations to simplify the equations (e.g., [8]) we work in a basis spanned by $|\nu_e\rangle$, $|\nu_x\rangle$, and $|\nu_y\rangle$:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = R_{23}^\dagger \begin{pmatrix} \nu_e \\ \nu_x \\ \nu_y \end{pmatrix},$$

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}. \tag{7}$$

Here and in the following, we use the shorthand $c_{kl} := \cos(\theta_{kl})$, $s_kl := \sin(\theta_{kl})$, $C_{kl} := \cos(2\theta_{kl})$, $S_{kl} :=$

$\sin(2\theta_{kl})$ for a mixing angle θ_{kl}. For the mixing angles we have the following values [9]: $\theta_{12} = 33.62°$, $\theta_{23} = 47.2°$, $\theta_{13} = 8.54°$.

Neutrinos are produced in flavor eigenstates so that their density matrices are diagonal. The off-diagonal ρ elements remain small unless something new happens in the form of self-induced flavor conversion caused by the neutrino-neutrino interaction. Matter effects in our case suppress vacuum flavor conversions. Thus, the self-induced oscillations can become large purely from the self-amplification which in turn requires instabilities (collective run-away solutions) in flavor space. To study these instabilities we can treat flavor correlations (off-diagonal terms of the density matrix) as plane waves $\rho_{ij} = Q_{ij}\, e^{-i\Omega t}$. This is the basis of the linearized stability analysis. If the frequency Ω has a nonzero imaginary part, then flavor conversions can occur driven by an exponentially growing factor.

After the evaluation of the commutators in Eq. (2) (see [9] for details on the Hamiltonian part and [6] for the dissipator's part) one can arrive at the following equation for eigenvalues for the four beam model with the decoherence term for the NO:

$$\left(\Omega + i\begin{pmatrix}\Gamma & 0 & 0 & 0\\ 0 & -\Gamma & 0 & 0\\ 0 & 0 & \Gamma & 0\\ 0 & 0 & 0 & -\Gamma\end{pmatrix}\right) Q = \left\{ |\omega| \begin{pmatrix} A+\eta B & 0 & 0 & 0\\ 0 & -A-\eta B & 0 & 0\\ 0 & 0 & A+\eta B & 0\\ 0 & 0 & 0 & -A-\eta B\end{pmatrix} + \lambda \begin{pmatrix}\Lambda & 0 & 0 & 0\\ 0 & \Lambda & 0 & 0\\ 0 & 0 & \Lambda & 0\\ 0 & 0 & 0 & \Lambda\end{pmatrix}\right.$$
$$+\,\mu\left[2\begin{pmatrix} g_{\bar l}\Lambda & 0 & 0 & -g_{\bar l}\Lambda\\ 0 & g_l\Lambda & -g_l\Lambda & 0\\ 0 & -g_{\bar r}\Lambda & g_{\bar r}\Lambda & 0\\ -g_r\Lambda & 0 & 0 & g_r\Lambda\end{pmatrix} + (1-\cos\alpha)\begin{pmatrix} g_{\bar r}\Lambda & -g_{\bar r}\Lambda & 0 & 0\\ -g_r\Lambda & g_r\Lambda & 0 & 0\\ 0 & 0 & g_{\bar l}\Lambda & -g_{\bar l}\Lambda\\ 0 & 0 & -g_l\Lambda & g_l\Lambda\end{pmatrix}\right.$$
$$\left.\left. +\,(1+\cos\alpha)\begin{pmatrix} g_l\Lambda & 0 & -g_l\Lambda & 0\\ 0 & g_{\bar l}\Lambda & 0 & -g_{\bar l}\Lambda\\ -g_r\Lambda & 0 & g_r\Lambda & 0\\ 0 & -g_{\bar r}\Lambda & 0 & g_{\bar r}\Lambda\end{pmatrix}\right]\right\} Q. \tag{8}$$

Here, $Q = (Q_r, Q_{\bar r}, Q_l, Q_{\bar l})^T$ is a 12 dimensional vector, where each Q_i consists of the three off-diagonal elements of the corresponding neutrino density matrix. To obtain a similar equation for the IO one can perform a simple substitution: $|\omega| \to -|\omega|$.

The matrices A and B in Eq. (8) contain combinations of mixing angles and come from the vacuum term (3) and read:

$$A_0 = \begin{pmatrix} -c_{12}^2 + c_{13}^2 s_{12}^2 & \frac{1}{2}S_{12}s_{13} & 0\\ \frac{1}{2}S_{12}s_{13} & C_{13}s_{12}^2 & \frac{1}{2}S_{12}c_{13}\\ 0 & \frac{1}{2}S_{12}c_{13} & c_{12}^2 - s_{12}^2 s_{13}^2\end{pmatrix},$$

$$B_0 = \begin{pmatrix} s_{13}^2 & 0 & 0\\ 0 & C_{13} & 0\\ 0 & 0 & -c_{13}^2\end{pmatrix}. \tag{9}$$

The matrix Λ originates from the matter part of the Hamiltonian (4) and is defined as $\Lambda = \mathrm{diag}(1,1,0)$. Γ is the decoherence matrix which in case of three neutrino flavors reads: $\Gamma = -\mathrm{diag}(\gamma_{21}, \gamma_{31}, \gamma_{32})$. For simplicity in our qualitative analysis we consider $\gamma_{21} = \gamma_{31} = \gamma_{32} = \gamma$.

The equation (8) has been solved numerically for the eigenvalues Ω with the neutrino–antineutrino and left–right asymmetry parameters $a = 0$ and $b = 0$ accordingly. In case of the relative angle $\alpha = 0$ we discover only the slow oscillation modes of two kinds:

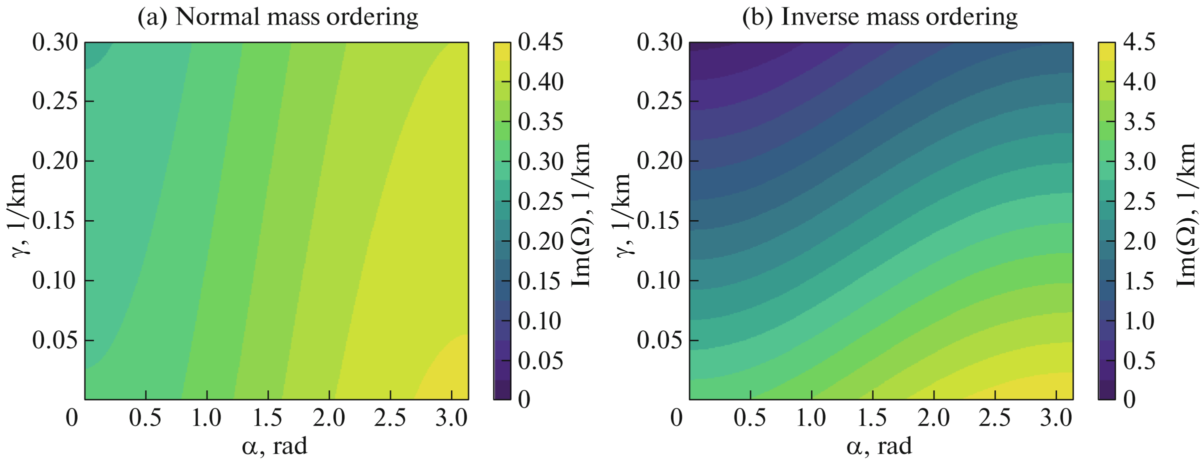

Fig. 1. Dependence of the slow symmetric instability $\mathrm{Im}(\Omega)$ on the relative angle α in the 4 beam model and decoherence parameter γ for both mass orderings. Here we have set a matter potential $\lambda = 10^2$ 1/km, a neutrino–neutrino interaction potential $\mu = 10$ 1/km, a vacuum oscillation frequency $\omega_{\text{vac}} = 0.015$ 1/km, a mass square ratio $|\eta| = 33$.

symmetric ones (solar) and antisymmetric (atmospheric). The growth rate of the latter is significantly larger than for the solar one. For $\alpha \in (0, \pi/2)$ an another faster growing mode appears.

For our purposes we have chosen to analyze the symmetric mode as it is persistent for all of the relative angles α and its characteristic value is rather small, so it's more prone to suppression. The results are shown in Fig. 1.

One can clearly see that the unstable mode resulting from a forward collision in the 4 beam model is suppressed for $\gamma \sim 0.3$ 1/km for the NO, while the unstable mode for the IO are still present. It is known that neutrino radiation field range from isotropic inside the proto-neutron star to being forward peaked at large distances (such as in the present study where $r \sim 10^2$ km) [10]. Given this distribution, we can assume that these collective modes might be effectively suppressed by the quantum decoherence in the case of the NO for essentially lower values of γ than for the IO. A similar behavior is present for asymmetric modes for overall larger value of γ.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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